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HIGHWAY RESEARCH RECORD

Number 422

Land Use and Transportation Planning

5 reports

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HIGHWAY RESEARCH RECORD

Number 422

Land Use and Transportation Planning

5 reports
prepared for the
51st Annual Meeting

Subject Areas

83 Urban Land Use
84 Urban Transportation Systems

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FOREWORD

This RECORD contains a series of papers that should be of interest to engineers and planners concerned with the theory and development of urban regions.

Bauer presents a descriptive analysis of subdivision activity in the southeastern Wisconsin region during a 50-year study period. The analysis is presented in 4 sections: subdivision platting activity by successive time periods, development patterns of the subdivisions, uses of the land within subdivisions, and platting activity related to sewerage service.

Brand reviews the theory of demand and its translation into current method in urban transportation planning, namely, the conventional sequence of urban travel-forecasting models. The current models are examined from the perspectives of appropriate structure, usefulness in practice, and relevance to emerging values.

Manheim presents some results in the theory of transportation system analysis. These results deal with properties of demand models. Although the author states that the results are largely theoretical, they have immediate practical application. The paper stresses the practical implications and only summarizes the key aspects of the theoretical results.

Dickey, Leone, and Schwarte present a technique for the optimum placement of activities in zones. The intent of the technique, initially developed for use in Melbourne, Australia, is to generate land use allocation schemes that are optimal according to some present objectives. The authors report on the use of the technique in Blacksburg, Virginia, to test a proposed land use scheme that became the basis for finding better arrangement patterns.

Sinha, Adamski, and Schlager discuss the problem of land use plan design and present support for a land use plan design model that can serve as a ready tool to achieve the optimal future plan for an urban area or a region through the satisfaction of the design cost units and at an optimum of public and private costs.

HISTORIC TRENDS IN LAND SUBDIVISION ACTIVITY IN SOUTHEASTERN WISCONSIN

Kurt W. Bauer, Southeastern Wisconsin Regional Planning Commission

•THE SOUTHEASTERN Wisconsin region, comprising 7 counties lying north of the Illinois-Wisconsin state line and west of the Lake Michigan shoreline, is one of the large urbanizing regions in the United States. The region has an area of 2,689 square miles, a population of 1.8 million people, and contains 153 general purpose local units of government, including Milwaukee, the twelfth largest city in the United States. The region, which contains 3 contiguous standard metropolitan statistical areas, as defined by the U. S. Bureau of the Census, is in many ways typical of most metropolitan areas of the United States.

Urban development has been taking place within southeastern Wisconsin since about 1840. For more than a century, this development occurred basically in the form of an outward expansion of the existing urban centers of the region and appeared in mapped form as a succession of narrow, concentric growth rings. From 1940 to 1950, urban development within the region continued to take place in a concentric pattern around the existing urban centers, but, in addition, fingers of development pointing outward from the larger central cities began to form. These fingers generally followed major highway routes, major stream valleys, and the Lake Michigan shoreline. This period also witnessed an intensification of urban development activity in the rural areas of the region, particularly around the shorelines of the many lakes.

From 1950 to 1970, a dramatic change occurred in the pattern of urban development. Contributing to this change were the pent-up housing demand present in the post-World War II years, an increase in automobile ownership and use, the accelerated construction of high-speed and all-weather highways, the widespread availability of electric power and telephone communication, the utilization of the septic tank as a means of on-site sewage disposal and of the shallow well as a means of on-site water supply, and the large-scale availability of relatively low-cost suburban land. The basic pattern of growth in evidence from 1940 to 1950 continued; but large, scattered tracts of rural lands were subdivided for urban use, and often equally large tracts of land were left between the old and the new development. This leapfrogging of development led to the use of the phrase "urban sprawl," defined as highly dispersed, low-density urban development. During this 20-year period, more than 190,720 acres of land were converted from rural to urban use within the 2,689-square-mile southeastern Wisconsin region, a 216 percent increase in such use. The population increase during this same period amounted to 515,468, a 42 percent increase. The overall population densities of the developed area of the region, which had peaked at about 11,500 persons per square mile in 1920, declined to about 8,500 persons per square mile in 1950 and to about 4,000 persons per square mile in 1970, or by about 53 percent.

The timing and spacing of land development within the region were examined in a historic platting study undertaken by the Southeastern Wisconsin Regional Planning Commission—the official area-wide planning agency for southeastern Wisconsin. Information was collected on quantity, character, rate, and geographic location of land sub-

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Sponsored by Group 1 Council.

division activity within the region during the 50-year period from 1920 through 1969. The study had the following objectives:

1. To determine the number of land subdivision plats recorded within the southeastern Wisconsin region since 1920 and to measure the amount of land committed to development in these plats;
2. To determine the temporal and the spatial distribution of land subdivision activity within the region since 1920 and since 1957 to determine the relation to sanitary sewerage service; and
3. To evaluate changing land subdivision design practices in terms of average subdivision size, average lot size, linear miles of streets created, type and amount of other dedicated lands, and chronological sequence and spatial distribution of various types of subdivision development patterns, such as grid, curvilinear, or cluster.

This paper presents a descriptive analysis of platting activity within the region for the 50-year study period. The analysis is presented in 4 sections: subdivision platting activity by successive time periods, development patterns of the subdivisions, uses of the land within the subdivisions, and platting activity related to sewerage service. The use allocation presents a discussion of the acreage actually devoted in the subdivision plats to lots or building sites, the area devoted to street right-of-way, and the area devoted to other dedicated uses such as park and school sites.

The Wisconsin Statutes require that for all land subdivisions an accurate plat be prepared by a registered land surveyor and recorded with the county register of deeds. The recorded plats provided the source of the data analyzed in this paper, a source that was unusually precise, accurate, complete, and reliable.

Within the southeastern Wisconsin region in 1970, urban land uses totaled 528.6 square miles, or 20 percent of the total area of the region. Of this 528.6-square-mile area, 244.2 square miles, or 46 percent, were devoted to residential use. The area of all residential plats studied totaled 146.9 square miles, or 60.1 percent of the total residential land area in the region.

PLATTING ACTIVITY IN THE REGION

In the 50-year period from 1920 through 1969, there were 4,907 residential subdivision plats recorded within the region. These subdivisions encompassed a total area of 94,050 acres and contained an average of about 19 acres each (Table 1). The most active period was the 1950-1959 post-World War II decade within which 1,797 plats, or 37 percent of the total, were recorded. The second most active period was the 1920 to 1929 predepression decade within which 1,367 plats, or 28 percent of the total, were recorded. Together these 2 periods accounted for nearly two-thirds of all residential subdivision plats recorded within the region since 1920, and the combined acreage platted during these 2 periods accounts for approximately 66 percent of the total acreage platted since 1920. The 1930-1939 depression decade, when only 215 plats were recorded, and the 1940-1949 World War II decade, when 444 plats were recorded, together account for less than 14 percent of the total plats recorded and for less than 12 percent of the total acreage platted since 1920. These 2 time periods also exhibit the lowest average subdivision plat area of 16.3 and 16.7 acres respectively.

PATTERNS OF DEVELOPMENT

Three residential subdivision patterns were identified for the purposes of the platting study on the basis of the predominant street layout used in the subdivision (Fig. 1):

1. The grid pattern has a predominance of straight streets intersecting at approximately right angles, is generally laid out approximately in the cardinal directions, has fairly uniform rectangular lots fronting on the gridiron streets, and often has alleys that provide a secondary means of access to the rear of each lot.
2. The curvilinear pattern has a predominance of curved streets, the locations of which have been adapted to the terrain, and frequently contains a variety of lot sizes and shapes fronting on loops, cul-de-sacs, through streets, and curvilinear streets.

3. The cluster pattern has a preponderance of groups or clusters of wedge-shaped lots around loop, cul-de-sac, and bulb streets and by open spaces called "commons" between these groups.

Grid Pattern

The most prevalent subdivision pattern within the region since 1920 has been the grid pattern, which accounted for 3,698 of the recorded subdivisions, or 75.4 percent of the total plats recorded. The 56,094 acres of land platted for such grid development accounted for 59.6 percent of the total acreage platted. The curvilinear pattern accounted for 1,203 recorded subdivisions, or 24.5 percent of the total plats recorded, and for 37,335 acres, or 39.7 percent of the total platted acreage. The cluster pattern of development, which is a more recent platting innovation within the region, accounts for less than 1 percent of either the number of plats recorded or the total area platted. Although the greatest number of grid subdivisions was recorded in the 1950's, the greatest amount of acreage was platted under the grid pattern during the 1920's. The average size of the grid-pattern subdivision has been decreasing since 1920.

Curvilinear Pattern

The 1950 decade accounted for the greatest number of curvilinear residential subdivisions and the greatest amount of acreage platted. The depression years from 1930 through 1939 and the war years from 1940 through 1949 combined accounted for less than 10 percent of the curvilinear subdivisions recorded and acreage platted. The average size of the curvilinear-pattern subdivision increased from 29.7 acres in the 1920's to 32.2 acres in the 1950's. A decrease in the average size during the 1960's reflects perhaps the development of smaller tracts of land that may have been bypassed during earlier periods. Although the average size of the grid subdivision has been decreasing, the average size of the curvilinear subdivision, with the exception of the 1960's, has been increasing. Moreover, since the 1950's, more acreage has been platted under the curvilinear pattern than under the grid pattern, even though there were fewer curvilinear subdivisions recorded in that period. Consequently, even though grid subdivisions account for the greater proportion of recorded subdivisions, the curvilinear subdivisions currently account for the greater amount of acreage platted.

Cluster Pattern

The cluster subdivision is only a very recent design innovation in southeastern Wisconsin. As of 1969, only 6 such subdivisions had been recorded. They encompass 621 acres and were all recorded in the 1960-1969 period. The most significant fact concerning such subdivisions is the large average size of approximately 103 acres.

USES OF LAND IN PLATTED AREA

The intended use of land within recorded residential subdivision plats fell into 3 major categories: residential building sites or lotted areas, dedicated areas, and nonlotted areas. Table 2 gives the number of acres set aside for these uses during each decade:

1. The lotted areas consist of recognizable land divisions identified as numbered lots specifically intended for residential building development.
2. The dedicated areas consist of land areas designated specifically for public or semipublic streets, alleys, pedestrian walks, other public ways, drainageways, schools, parks, commons, buffer zones, planting strips, sites for utility facilities (such as water storage tanks and sewage pumping stations), and sites for various types of recreational uses (such as bridle paths, boat landings, beaches, or water channels and impoundments).
3. The nonlotted areas consist of land divisions not designated for some specific public or semipublic use and apparently not intended, at the time of recordation, for development into residential building lots. Many of these areas were designated simply as "outlots," and most of them obviously represented remnants of land that the road pattern or lotting pattern rendered inaccessible or unusable as building sites at the

Table 1. Residential subdivisions recorded and platted.

		Total					Grid Pattern							Curvilinear Pattern						
		Subdivisions			Acres Platted		Subdivisions				Acres Platted			Subdivisions				Acres Platted		
								Percent				Percent			Percent				Percent	
Decade	Population[a]	Number	Percent	Avg Size (acres)	Number	Percent	Number	of Total	of Grid	Avg Size (acres)	Number	of Total	of Grid	Number	of Total	of Curvi.	Avg Size (acres)	Number	of Total	of Curvi.
1920	783,681	1,367	27.9	21.0	28,726	30.6	1,227	89.8	33.2	20.0	24,569	85.5	43.8	140	10.2	11.6	29.7	4,157	14.5	11.1
1930	1,006,118	215	4.4	16.3	3,509	3.7	188	87.4	5.1	14.3	2,682	76.4	4.8	27	12.6	2.2	30.6	827	23.6	2.2
1940	1,067,699	444	9.0	16.7	7,435	7.9	356	80.2	9.6	13.1	4,660	62.7	8.3	88	19.8	7.3	31.5	2,775	37.3	7.4
1950	1,240,618	1,797	36.6	18.7	33,603	35.7	1,268	70.6	34.3	13.1	16,594	49.4	29.6	529	29.4	44.1	32.2	17,009	50.6	45.6
1960	1,573,620	1,084	22.1	19.2	20,777	22.1	659	60.8	17.8	11.5	7,589	36.5	13.5	419	38.7	34.8	30.0	12,567	60.5	33.7
Total		4,907	100.0	19.2	94,050	100.0	3,698	75.4	100.0	15.2	56,094	59.6	100.0	1,203	24.5	100.0	31.0	37,335	39.7	100.0

Note: During the 1960's, there were 6 cluster residential subdivisions, or 0.5 percent of the 1960 total and 0.1 percent of the overall total. They included 621 platted acres, or 3.0 percent of the 1960 total and 0.7 percent of the overall total.

[a]At the beginning of the decade. Regional population in 1970 was 1,756,086.

Figure 1. Subdivision development patterns.

Table 2. Acreage in residential subdivision land use categories.

			Dedicated																	
							Other													
	Lotted[a]		Streets[a]		Alleys[a]		Total[a]		Parks[b]		Recreation[b]		Schools[b]		Miscellaneous[b]		Nonlotted[a]			
Decade	Number	Percent	Number	Percent	Number	Percent	Number	Percent	Number	Percent	Number	Percent	Number	Percent	Number	Percent	Number	Percent		
1920	20,006	69.6	6,832	23.8	491	1.7	474	1.7	264	55.7	179	37.8	12	2.5	19	4.0	923	3.2		
1930	2,595	74.0	748	21.3	40	1.1	25	0.7	19	76.0	6	24.0	—	—	—[c]	—[d]	101	2.9		
1940	5,526	74.3	1,475	19.8	28	0.4	148	2.0	101	68.2	25	16.9	6	4.1	16	10.8	258	3.5		
1950	24,713	73.5	7,377	22.0	89	0.3	433	1.3	236	54.5	37	8.5	39	9.0	121	28.0	991	2.9		
1960	15,419	74.2	4,207	20.2	8	—[d]	530	2.6	156	29.4	161	30.4	4	0.8	209	39.4	613	3.0		
Total	68,259	72.6	20,639	21.9	656	0.7	1,610	1.7	776	48.2	408	25.3	61	3.8	365	22.7	2,886	3.1		

[a]Percentage is of total area platted that decade. [b]Percentage is of other dedicated areas platted that decade. [c]Less than 0.5 acre. [d]Less than 0.05 percent.

Table 3. Characteristics of platted residential lots.

	Total						Grid Pattern					Curvilinear Pattern				
Decade	Number	Percent	Avg[a]	Width (ft)	Depth (ft)	Area (ft^2)	Number	Avg[a]	Width (ft)	Depth (ft)	Area (ft^2)	Number	Avg[a]	Width (ft)	Depth (ft)	Area (ft^2)
1920	155,658	53.0	114	41	125	5,125	140,405	115	40	123	4,920	15,253	109	55	142	7,810
1930	10,833	3.7	50	59	152	8,968	8,382	45	58	152	8,816	2,451	91	64	154	9,856
1940	17,696	6.0	40	73	165	12,045	11,339	32	73	165	12,045	6,357	72	75	166	12,450
1950	72,090	24.7	40	86	155	13,330	40,472	32	86	155	13,330	31,618	60	96	163	15,648
1960	37,776	12.8	35	97	160	15,520	18,075	27	83	145	12,035	19,068	46	111	175	19,425
Total	294,053	100.0	60	62	140	8,680	218,673	59	53	133	7,049	74,747	62	88	162	14,256

Note: Width, depth, and area are dimensions of a typical lot. A lot was selected as being typical of each subdivision, and widths and depths were weighted, averaged, and then multiplied to yield the typical dimensions for each time period.

[a]Per subdivision.

time of initial subdivision. Many of the larger areas in this category were found to have been further subdivided into building sites through "replatting" at later dates.

Lotted Areas

The 4,907 residential subdivisions recorded during the 50-year period contained 294,053 residential lots (Table 3) covering 68,259 acres. During the 1920's, 53 percent of the total number of lots were platted and 20,006 acres of residential land were set aside for lots, but that acreage is less than one-third of the 68,259 acres set aside for lots during the entire 50-year period. The 1950's accounted for 37 percent of all plats recorded, 36 percent of all the acreage platted, 37 percent of the residential acreage platted, but 25 percent of the lots created. This suggests a subdivision design trend toward larger average lot sizes.

The proportion of residential land within the recorded plats has not varied appreciably during the entire period. It reached a low of 69.6 percent in the 1920's and a high of 74.3 percent in the 1940's (Table 2). The average during the entire study period was 72.6 percent.

The timing and spatial distribution of land development are the most important factors influencing efficient and economical provision of public services. Closely linked to these factors are subdivision size and lot size. The size of a residential lot will greatly influence not only type, style, and price range of the structure to be placed on the site but also quantity and quality of public services that can be economically provided to the site. Typical lot sizes have been increasing steadily since the 1920's, when the typical lot contained approximately 5,125 ft^2 (Table 3). By the 1960-1969 period, the typical lot had more than tripled in size and contained approximately 15,520 ft^2. During the entire study period, the typical lot dimensions have also changed significantly, principally in the front footage dimensions. During the 1920's, when most of the lots were created, a typical lot measured approximately 40 ft wide by 125 ft deep. In the 1960's, a typical lot measured approximately 97 ft wide by 160 ft deep. As the average lot size increased, the average number of lots per platted subdivision decreased, dropping from an average of 114 lots per subdivision in the 1920's to an average of 35 lots per subdivision in the 1960's.

Grid Pattern—The grid pattern of residential subdivision accounted for the creation of 218,673, or 74 percent, of the lots platted within the region during the 50-year study period. The grid pattern of development also shows a steady decrease in the average number of lots created per subdivision and a gradual increase in the typical lot size until the 1960's when there was a slight decrease. The average front footage or width of the typical lot increased from 40 ft in the 1920's to 83 ft in the 1960's.

Curvilinear Pattern—The curvilinear pattern of residential subdivision accounted for the creation of 74,747, or about 25 percent, of the lots platted within the region during the study period. The average number of lots per recorded subdivision decreased, but the average size of the typical lot increased. Most of the change in lot area is attributable to an increase in the lot width from 55 ft in the 1920's to 111 ft in the 1960's.

Cluster Pattern—Only 6 cluster subdivision plats were recorded, and these 6 plats accounted for the creation of 633 residential lots. The typical cluster pattern of development contained an average of 106 lots measuring approximately 96 ft in width and 139 ft in depth. The typical lot area was approximately 13,300 ft^2, which, as intended by this design type, represents a smaller average lot size than the curvilinear pattern of development.

Area Dedicated for Streets

During the study period, there were 20,639 acres, or 22 percent, of land dedicated for street right-of-way through the recordation of plats (Table 2). The average subdivision contained 4.2 acres of land so dedicated (Table 4). The street centerline measurement of the dedicated rights-of-way totaled 2,837 linear miles, an average of 0.7 mile per recorded subdivision. The greatest number of acres for street right-of-way was dedicated in the 1950's, but the greatest number of linear miles was created

during the 1920's. Of the total land platted in the 1920's, 23.8 percent was dedicated to streets. During that same period, more than 33 percent of the grid pattern of residential subdivisions was recorded. These subdivisions rely heavily on the uniform and regular street pattern as a design element. The proportion of land dedicated for street right-of-way within the recorded subdivision plats has decreased from 23.8 percent in the 1920's to 20.2 percent in the 1960's, a significant reduction in such an important subdivision design element as streets.

Grid Pattern—In grid subdivisions, the 1920-1929 period was the most active in terms of acreage recorded and linear miles of street right-of-way and acreage dedicated; however, the greatest number of grid subdivisions was recorded in the 1950-1959 period. The linear miles of street right-of-way dedicated in subdivision plats have decreased consistently during the study period, but the proportion of acreage dedicated for street right-of-way has remained relatively stable. This suggests a growing tendency for wider street right-of-way to be dedicated.

Curvilinear Pattern—The 1950-1959 period was the most active period for recording curvilinear subdivisions. This period also accounted for the largest average street right-of-way acreage per recorded plat and for nearly half of the total linear miles of streets created within curvilinear subdivisions during the study period.

Cluster Pattern—The 6 cluster subdivision plats encompassed 621 acres of land, of which 101 acres, or 16.3 percent, were dedicated for street right-of-way, an average of 16.8 acres, or 2.4 linear miles per recorded subdivision.

Area Dedicated for Alleys

Only 1,052 subdivisions, or 21 percent of the total, contained land area dedicated for alleys. Moreover, the proportion of land dedicated for alley rights-of-way during the study period decreased from 1.7 percent of the total acreage recorded in the 1920-1929 period to less than 0.1 percent in the 1960-1969 period. More than 90 percent of the alley right-of-way was located in grid subdivisions. It is evident that the alley, once considered essential to the design layout of a subdivision, is now rarely incorporated into a subdivision layout.

Area Dedicated for Uses Other Than Streets and Alleys

Approximately 1,610 acres of land within platted residential subdivisions, or 1.7 percent of the total area, were dedicated for purposes other than street and alley rights-of-way. The uses of this acreage are given in Table 2. More than 51 percent of the land dedicated for purposes other than streets and alleys within platted subdivisions was located in curvilinear subdivisions, 37 percent was located in grid subdivisions, and 12 percent was located in cluster subdivisions. Moreover, 60 percent of the land dedicated for parks, 39 percent of the land dedicated for other recreation uses, and 67 percent of the land dedicated for schools was located in curvilinear subdivisions. Not all platted residential subdivisions contained land areas dedicated for public uses. As a matter of fact, only 740, or about 15 percent, of the 4,907 subdivisions recorded during the study period contained land areas dedicated for purposes other than streets and alleys, and many of these contained areas dedicated for several types of uses, such as schools, parks, and pedestrian ways.

Nonlotted Areas

Table 2 gives the nonlotted acreage of recorded plats since 1920; the proportion remained fairly constant throughout the study period. Of this acreage, 58 percent was located in grid subdivisions, 37 percent was located in curvilinear subdivisions, and 5 percent was located in cluster subdivisions.

PLATTING ACTIVITY RELATED TO SANITARY SEWERAGE SERVICE

The 1955 Wisconsin Legislature revised the Wisconsin Platting Act. One of the significant revisions required subdivision plats not served by sanitary sewers to be approved by the Wisconsin Board of Health. This revision permitted an evaluation of

Table 4. Linear miles and acreage of dedicated street right-of-way.

	Total					Grid Pattern					Curvilinear Pattern				
	Miles[a]		Acres			Miles		Acres			Miles		Acres		
Decade	Number	Avg[b]	Number	Percent	Avg[b]	Number	Avg[b]	Number	Percent[c]	Avg[b]	Number	Avg[b]	Number	Percent[c]	Avg[b]
1920	1,027.3	0.8	6,832	23.8	5.0	885.0	0.7	5,901	24.0	4.8	142.3	1.0	931	22.4	4.5
1930	102.6	0.5	748	21.3	3.5	76.4	0.4	569	21.2	3.0	26.2	1.0	179	21.6	4.6
1940	204.7	0.5	1,475	19.8	3.3	127.1	0.4	911	19.5	2.6	77.6	0.9	564	20.3	4.9
1950	958.8	0.5	7,377	22.0	4.1	493.9	0.4	3,789	22.8	3.0	464.0	0.9	3,588	21.1	4.7
1960	543.6	0.5	4,207	20.2	3.9	224.5	0.3	1,718	22.6	2.6	304.8	0.7	2,388	19.0	5.3
Total	2,837.0	0.6	20,639	21.9	4.2	1,806.9	0.5	12,888	23.0	3.5	1,015.8	0.8	7,650	20.5	4.9

[a]Based on measurement of centerline of dedicated right-of-way. [b]Per subdivision. [c]Of acreage recorded in that pattern.

Table 5. Residential plats and lots recorded with and without public sewerage facilities available —1957-1969.

	Plats						Lots					
	Total		With		Without		Total		With		Without	
Pattern	Number	Percent	Number	Percent	Number	Percent	Number	Percent	Number	Percent	Number	Percent
Grid	995	61.5	828	68.7	167	40.5	26,752	46.5	23,202	51.8	3,550	27.8
Curvilinear	616	38.1	372	30.9	244	59.2	30,201	52.4	21,008	46.9	9,193	71.9
Cluster	6	0.4	5	0.4	1	0.3	633	1.1	594	1.3	39	0.3
Total	1,617	100.0	1,205	100.0	412	100.0	57,586	100.0	44,804	100.0	12,782	100.0

the subdivision platting activity that took place during the 1957-1969 period with respect to the provision of sanitary sewerage service. Although there is no question that many recorded subdivisions were developed prior to 1957 without provision for sanitary sewerage facilities, records concerning this fact were not required and thus were not available for the years prior to 1957.

During the period from 1957 through 1969, there were 1,617 residential subdivisions encompassing 30,051 acres recorded within the region (Table 5). Nearly three-fourths of those recorded were provided with public sanitary sewerage facilities, and more than one-fourth were provided with on-site soil absorption sewage disposal systems for the lots platted. During this period, 59 percent of the recorded subdivisions that had no public sewerage provided were of the curvilinear pattern and about 41 percent were of the grid pattern. Many subdivisions that had no public sanitary sewerage facilities available at the time of recordation and initial development have subsequently been provided with such public facilities, often at considerable additional cost to both the private property owners and the local units of government concerned.

More than 77 percent of the lots created during the 1957-1969 period were provided with public sewerage facilities, and nearly 52 percent of these were in grid subdivisions. About 22 percent of the lots were not provided with public sewerage facilities, and nearly 72 percent of these were located in curvilinear subdivisions.

SUMMARY AND CONCLUSIONS

For more than 100 years, from 1840 to 1950, urban development took place within the southeastern Wisconsin region by a generally continuous outward expansion of the urban centers established early in the settlement of the region. From 1950 to 1970, however, a dramatic change occurred in this pattern of urban development: Large, scattered tracts of rural lands were subdivided for urban use and resulted in a highly dispersed, discontinuous, low-density development pattern that has become known as urban sprawl. To provide information on the changes brought about by the location and timing of this urban land development process, a study was undertaken by the Southeastern Wisconsin Regional Planning Commission of the quantity, character, rate,

and geographic location of residential land subdivision activity within the region during the 50-year period from 1920 through 1969. A review of the changes in the Wisconsin platting laws governing the subdivision of land over this period was also conducted. The findings of the study are summarized below.

Total Subdivisions

The 50-year period witnessed the recordation within the southeastern Wisconsin region of 4,907 residential subdivision plats encompassing 94,050 acres of land and accounting for the creation of 294,050 residential lots and the dedication of 20,639 acres of street right-of-way, 656 acres of alley right-of-way, 776 acres of park land, 408 acres of other recreation land, and 61 acres of land for school purposes. More than half of the subdivisions recorded and of the acreage platted during the 50-year period were recorded and platted since 1950, but only 35 percent of the lots were created since 1950, indicating both a trend toward larger lots and increased platting activity during the past 20 years. Although the acreage of the average subdivision remained nearly constant during the 50-year period, the number of lots per subdivision decreased from an average of 114 lots per subdivision in the 1920's to an average of 35 lots per subdivision in the 1960's. The typical lot size increased from approximately 5,100 ft^2 in the 1920's to approximately 15,500 ft^2 in the 1960's.

Grid Subdivisions

The grid pattern was used for 3,698 subdivisions, or 75 percent of the total recorded, and accounted for 56,094 acres, or 60 percent of the total acreage platted. In those subdivisions, 218,673 residential lots, or 74 percent of the total number, were created, and about 1,800 linear miles of street right-of-way, or nearly 64 percent of the total, were dedicated. Although most subdivisions platted within the region were of the grid pattern, the proportion has decreased steadily since 1920. Since 1950, the grid subdivision has accounted for less than half of the total acreage platted. Also during the 1960's, lots in grid subdivisions accounted for fewer than half of the total lots created, indicating a decreased reliance on the grid pattern for land subdivision. Moreover, the average size of the grid subdivision has also been decreasing since 1920, as has the average number of lots per subdivision. In the 1920 through 1929 period, the average grid subdivision was 20 acres in size and contained an average of 115 lots. By the 1960's, the average grid subdivision was 11 acres and contained an average of only 27 lots. The typical grid subdivision has followed the trend indicated earlier in that the typical lot area has increased from approximately 5,000 ft^2 in the 1920's to approximately 12,000 ft^2 in the 1960's. The principal change affecting the lot area occurred in the typical lot frontage, which increased from 40 ft to approximately 80 ft during the 50-year period.

Curvilinear Subdivisions

During the study period, the curvilinear residential subdivision design was used for 1,203 subdivisions, or nearly 25 percent of the total recorded, and accounted for 37,335 acres, or nearly 40 percent of the total acreage platted. In those subdivisions, 74,747 residential lots, or 25 percent of the total number, were created and 1,016 linear miles of street right-of-way, or nearly 36 percent of the total street mileage, were dedicated. The curvilinear pattern accounted for 10 percent of the recorded subdivisions, 15 percent of the platted acreage, and 10 percent of the lots created in the 1920's. By the 1960's, the curvilinear pattern accounted for 39 percent of the recorded subdivisions, 60 percent of the platted acreage, and 52 percent of the lots created. The average size of the curvilinear subdivision has increased from 29 acres in the 1920's to 30 acres in the 1960's. The average number of lots per subdivision, however, has decreased from 109 in the 1920's to 46 in the 1960's. The typical lot area has increased from approximately 7,800 ft^2 in the 1920's to 19,400 ft^2 in the 1960's. The principal change affecting the lot area was the lot frontage, which increased from 55 ft in the 1920's to about 110 ft in the 1960's.

Cluster Subdivisions

The cluster residential subdivision design has been introduced into the region only since 1960 and has been used for only 6 subdivisions, or less than 1 percent of the total recorded. It accounted for 621 platted acres, or less than 1 percent of the total acreage platted; created 633 residential lots, or less than 1 percent of the total lots created; and resulted in the dedication of 14 linear miles of street right-of-way, or less than 1 percent of the total street mileage dedicated. Although it has had only a limited application in the region to date, the average cluster subdivision contains about 103 acres, an average of 106 lots, and a typical lot area of approximately 13,300 ft^2.

Provision of Sewerage Service

A special effort was made to determine the amount of platting activity that took place outside established public sanitary sewerage service areas; this revealed that 412 recorded plats, or 40 percent of the total number platted during the entire period, required review and approval by the Wisconsin Board of Health because no provisions were made for public sanitary sewerage service to the lots created. Moreover, of these 412 subdivisions, 240, or 58 percent, were located in quarter sections within which more than 50 percent of the area was covered by soils having severe or very severe limitations for such residential development. Of those 240, 131 were located in quarter sections wherein the entire land area was covered by soils having severe or very severe limitations for residential development requiring on-site sewage disposal.

THEORY AND METHOD IN LAND USE AND TRAVEL FORECASTING

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This paper reviews the theory of demand and its translation into current method in urban transportation planning, namely, the conventional sequence of urban travel-forecasting models. The current models are examined from the perspectives of appropriate structure, usefulness in practice, and relevance to emerging values. The models appear to faithfully reflect the understanding of land use location and travel behavior and of the information requirements of an earlier period in urban transportation planning. A transportation-related general equilibrium land use model is derived, based on a causal theory of travel, namely, the theory of urban person travel as a derived demand. This long-run, activity-distribution, general model is used to examine the new set of (short-run) travel-demand models employing direct and cross relations and then the conventional sequence of traffic models: trip generation, trip distribution, and modal split. The simplifying assumptions required for these models are explicitly examined for their structural (causal) and statistical implications. It is concluded that separate modeling of short-run travel demand from long-run activity location introduces structural and statistical problems whose implications require further research. However, the structural and specification errors revealed in the current conventional models are such that they are of doubtful validity and produce possibly misleading travel forecasts. Such forecasts are in danger of being bypassed in current urban transportation controversies, and consideration of user travel costs may be bypassed with them.

•IN THEORY, demand is a function, not a fixed quantity. Demand models relate quantities of travel demanded to resources expended by travelers. The latter are travel times and costs, broadly defined, incurred on or supplied by the transportation system. How accurately and usefully has this theory of demand been translated into method in urban transportation planning? This paper first examines the current method, namely, the conventional sequence of travel-forecasting models. The current method is examined from the standpoint of appropriate structure and usefulness in current and emerging practice in urban transportation planning.

CURRENT PREDICTIVE MODELS IN URBAN TRANSPORTATION PLANNING

Current practice in predicting quantity of travel on transportation networks is based on the theory of equilibrium between supply and demand on the transportation network. That is, there should be an equality between the travel conditions found (such as times and costs) on the loaded network and the travel conditions used as input to the prediction. The current well-known conventional procedure is to model travel behavior as a series of sequential, independent choices of trip generation, trip distribution, modal split, and traffic (route) assignment. Land use forecasting precedes travel forecasting

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as a separate step. For each travel choice, the existing pattern of usage in the region at the prevailing equilibrium between supply and demand is related to a small set (often one) of independent variables. The trend or description is then assumed to hold in the future.

For example, trip distribution is modeled as a function of a simple description of the trip lengths that prevailed at the equilibrium between supply and demand represented in the base-date data file. The usual trip-generation procedure relates total trips in and out of a zone only to measures of the activities existing in the zone. The assumption is made that total travel, as measured by trip ends, varies only as development varies, not as conditions on the tested networks change.

In addition, there are computational and logical difficulties in bringing the predicted travel conditions into line with the conditions (if any) used as input to each of the component travel-choice models. There is no assurance that travel times and costs resulting from traffic assignment will equal travel times and costs explicitly or implicitly input into each sequentially applied model of component travel choice (1)—that is, that an internally consistent network equilibrium will be produced.

One may reflect that the urban transportation studies in the 1950's and 1960's took the easy way out by equating usage (a constant) with demand in calibrating their models. For existing conditions, the models fit well with usage. Not generally recognized was that present usage is merely a fixed quantity of travel demanded at existing levels of supply, accessibility, and benefits from opportunities at existing trip ends. The simple trends or descriptions contained in the conventional models cannot be predicted forward with much confidence in a situation as complex as travel within an urban region.

The shortcomings of the conventional models increase when predictions are made of travel on congested networks (i.e., when small changes in assigned link travel volumes result in large changes in link travel times and delay). Since large-capacity, relatively congestion-free expressways in high-density urban areas are increasingly difficult (if not impossible) to build in the era of urban highway controversies (2), we can look forward to the future equilibrium between supply and demand being quite different from that which existed in the early 1960's when most of our large-scale transportation study data collection took place. Society's changing values introduce new conditions and information requirements in the transportation modeling process.

Operationality in transportation planning today requires demonstrating how smaller transportation systems accommodate smaller amounts of travel and how greater systems accommodate greater amounts of travel. Savings in resources expended by travelers (i.e., user benefits) from transportation improvements must be accurately calculated and vary appropriately with the total resources expended by society to provide those benefits. The latter resources, which are increasingly highly valued by society, include air and noise pollution, safety, community disruption, and many other effects that are external to the calculation of travel demand in a predictive model. Accurate travel forecasts are needed to calculate their magnitudes.

Only by explaining the causal relations underlying travel demand can accurate forecasts be made of future changes in the performance of a transportation system as land uses and transportation facilities change. Emerging values and information requirements of transportation decision-makers require policy-sensitive demand models in transportation planning.

New theory that improves our understanding of travel behavior can help in structuring appropriate travel-demand models. The theory also helps identify the important variables that affect individual travel decisions and that should be included in the models. The derivation of a "new" method in the next section proceeds from a new theory of travel demand.

DERIVING A GENERAL LAND USE MODEL

A recent major theoretical paper on the subject of travel-demand forecasting defines passenger travel as a derived demand: "A trip is made because a household member wishes to purchase commodities or services, or obtain other satisfactions such as the purchase of food, a visit to the doctor, or obtaining of income (through work)." How-

ever, the papers of Kraft and his colleagues (3, 4), which contain this and other fundamental contributions to travel-demand theory and behavioral modeling, do not model travel explicitly as a derived-demand commodity. An extension of their modeling into the area of land use or general equilibrium modeling is presented here. In the process, the theory of travel as a derived demand can be incorporated explicitly into the more general model. This is made possible (or at least made easier) because the larger general model permits travel to be modeled as one intermediate (derived) output of the larger urban system.

The land use or general model once derived can be altered by making some simplifying assumptions in a way that produces the earlier behavioral travel forecasting models. In addition, the general model can be simplified still further to produce each of the current conventional travel models, namely, trip generation, trip distribution, and modal split. In the process of this successive alteration of the general model, the simplifying assumptions in (short-run) travel-demand models in general, and in the current conventional models in particular, are clearly illustrated. The implications of these assumptions can then be examined.

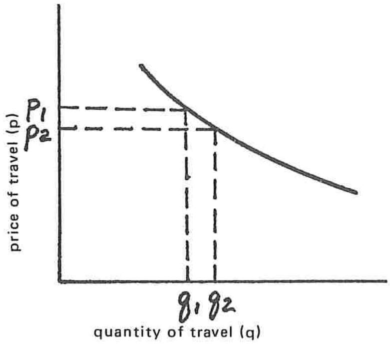
Theory: The General Model

Because travel is a derived demand, as defined above, trips would not be made if the benefits to be derived at their destinations were not greater than the resources expended in getting to and from the destinations. This holds whether we consider one round trip or whether we consider tours, that is, trips involving multiple stops. [There is increasing evidence indicating the importance and prevalence of tours (5). Intermediate legs of such tours are modeled separately as non-home-based trips in the conventional models. Much useful information on preceding mode and the like is lost by not modeling these trips as tours.] The base location can be considered an arbitrary zero or reference point where the benefits from consumption of the output of the activity at that location is less than at most other locations. (The most logical base point is an open question. Home is usually taken as the base. However, everybody may be destined for home because consumption of activities at home is valued most highly. The theory is independent of this problem.)

In the case of urban passenger travel, we can define the resources expended by consumers of travel in the usual way, namely, the traveler's money and time. (Time here is activity specific. Its value, relative to money and to consumption of outputs at trip ends, is dependent on the activity engaged in during travel, i.e., the method of travel and its component parts.) If the output at the trip destination is valued more highly by the traveler than the resources expended in travel, he will make the trip. The difference between the two (if any) is the net benefit of the trip.

The existing universe of travel and activities in a region represents some equilibrium between the preferred activities of residents and the desire to minimize resources in travel. (This is not a static equilibrium in reality, nor are the activities engaged in or the resources expended in travel intended to be described here as optimal or minimal in any way.) If improvements in transportation result in some lowering of the resources that must be expended per unit of travel, we can assume there will be an equal or greater amount of travel consumed by the individual, or output by the transportation system. (Travel must be consumed to be produced: an interesting and known identity.) That is, new opportunities at trip destinations farther away in distance but not in travel cost will come into range, and they will offer an increased net benefit from travel—often, of course, at the "expense" of previous destinations! (Eventually the "quantity" of travel must be operationally defined. Distance is a useful interpretation at this stage because the producer's cost of supplying or outputting travel is logically related to distance.) Thus, improvements in transportation that lower the cost of travel (i.e., money and time resources that must be expended in travel) tend to alter the former equilibrium. Note that only monotonic behavior is assumed; that is, increases in travel do not necessarily result from transportation improvements.

We can draw a monotonic curve that is purely descriptive of this covariation in the price of travel output by the transportation system and the quantity of travel consumed in the region:



As the price of travel (the resources in time and money expended in travel) decreases from p_1 to p_2, the quantity of travel in the region is equal to or greater than its previous amount ($q_2 \geq q_1$, and $dq/dp \leq 0$).

We note here only the most simple linear equation that describes the curve graphed above:

$$q = a + bp$$
$$b \leq 0$$

Inasmuch as we assume that people minimize their costs or resources expended in travel in order to maximize their net benefits from travel, we can assume that people tend to choose their methods of travel (mode, route, and time during the day) to minimize their costs or resources expended in travel. If some alternate method of producing travel (e.g., alternate mode or route) presents itself that involves lower cost to the traveler, we can assume that the traveler will choose that alternative in order to maximize the net benefit from travel. The closer the substitute is, the greater the switch will be from one alternative to the other. Also, the greater the cost savings on the alternate are, the greater the tendency will be to increase the quantity of travel on the substitute (and to increase total travel and thus net benefits from travel). Also, higher cost savings on the alternate will decrease the quantity of travel by the first method.

This behavior can again be described in simple linear travel-method-specific (e.g., mode) equations employing direct and cross relations:

$$q_1 = a_1 + b_1p_1 + c_1p_2$$
$$q_2 = a_2 + b_2p_2 + c_2p_1$$
$$b_i \leq 0$$
$$c_i \geq 0$$

where 1 is the first travel method and 2 is its substitute.

Defining exactly or precisely the commodities and the relevant market so that useful direct and cross relations may be developed is an important problem in demand analysis. High cross relations between commodities are indicative of a well-defined market and help to delimit the market (6). In our case we are fortunate in that there appears to be an identifiable and reasonably well-circumscribed market called urban travel. It remains to appropriately define the commodities making up that market.

The lumpiness of urban transportation technology, composed as it is of generally easily distinguishable (from the supply side and thus of great interest to those who must provide it) modes and links making up routes and networks, helps us (from the supply side) to distinguish among different methods of travel. Also, the value that individuals attach to component times and costs required to be expended on travel by the various available (thusly) defined substitute methods in an urban area appears logically and empirically to vary (4). Thus, the well-defined market, the variation in activity-specific time and money value, and the importance of the different travel methods to planners (from the supply side) suggest that we can profitably search for high cross relations of travel on alternate modes and routes and at alternate times.

In the simple equations presented thus far, there are important missing variables that describe how the system of interest behaves according to our theory. One missing variable is the output obtained from activities at the trip destinations. If the output (opportunities) at trip destinations in the region increases, there will be a tendency to increase the amount of travel in order to increase total net benefits from travel. At the new equilibrium, the increased value of outputs obtained at the trip ends would be equal to or greater than the increased resources expended on travel. This behavior can again be described in simple linear equations that are consistent with previous work (however, the present model, based on the theory of travel as a derived demand, is as yet incomplete):

$$q_1 = a_1 + b_1p_1 + c_1p_2 + d_{1k}A$$

$$q_2 = a_2 + b_2p_2 + c_2p_1 + d_{2k}A_k$$

$$d \geq 0$$

where A_k = measures of activities $(1, \ldots, k, \ell, \ldots, K)$ at the trip ends from which value is obtained.

Because additional travel is being traded off or expended in order to obtain higher valued outputs at the trip ends, we can expect the signs of the d coefficients to be positive. That is, higher valued activities can be expected to occur (covary) with greater amounts of travel.

General Equilibrium Model

We have defined travel as a derived-demand commodity. That is, it is desired not for its own sake, but as something on which resources must be expended in order to obtain the benefits of some output from activity at trip destinations. Therefore, the appropriate way to forecast a derived demand is to forecast the demand for the final good, namely, the trip-end outputs. The resources expended on travel (volume times cost) will be one of the costs of obtaining the final goods and, thus, it will appear in the predictive equations modeling demand for the final outputs (activities).

The equilibrium equations for the final outputs or activities in their simplest form for a region, again with 2 travel methods, are

$$A_k = a_k + b_kp_1 + c_kp_2 + d_{k\ell}A_\ell$$

$$b \leq 0$$

$$c \leq 0$$

For a region with an unspecified number M of alternate travel methods, the more general form of the linear model is

$$A_k = a_k + b_{km}p_m + c_{k,n\neq m}p_{n\neq m} + d_{k\ell}A_\ell$$

where m = method of travel $(1, \ldots, m, n, \ldots, M)$.

Some important changes take place in the signs on the coefficients in the general model for the (now causal!) price of travel variables from those in the previous descrip-

tive or covariational travel equations. The b and c coefficients will now both be negative. This is, of course, consistent with classical economic location theory (1). That is, land value (where land value is value in the long run, though presumably closely and directly related to the sum of short-run outputs of value to travelers) increases as transportation improvements are made that lower the price of travel to and from the location of the land.

This is conceptually a general equilibrium model in that its equations describe a set of conditions that when solved satisfy the stated conditions for the system to be in a state of general or static equilbrium. The equations are solved as a simultaneous set. The number of equations equals the number of activities K being forecast. An initial formulation of the activity variables for transportation study modeling involves measures of activities that relate closely to value obtained from travel, for example, employment, value of wages earned, retail sales, residential value (such as lot size), and measures of recreation and social potential.

A one-dimensional formulation of the transportation price variables involves relatively simple one-dimensional travel-method-specific accessibility (price vector) potential functions. In the two-dimensional case, the activity output variables would still be one dimensional (A_{ik}, i = location index), but the transportation price variables put into the model would be origin-destination and travel-method-specific price vectors (i.e., including components of price and service). The number of these variables could get quite large, of course, because travel method can incorporate mode, route, time of day, and other characteristics. Some form of the more general two-dimensional model is recommended for initial testing. Simplifications of the model will fall out as conclusions from model tests.

The theory of derived urban passenger travel demand does not fully circumscribe the entire set of causal relations among all factors in a city and all measures of activity location and intensity. That is, this is not a "complete" urban system model incorporating all possible causal relations operating in a region and influencing urban development patterns. However, such a complete model would probably lack practical usefulness in most applications.

According to modeling theory, we seek to isolate the important variables and their relations in a purposeful way that contributes to the analysis and, in this case, that implements operationality in transportation planning. Thus, transportation as a causal determinant of activity distribution changes is stressed. The general model allows an improved understanding of the simultaneous determination of travel and activity patterns.

EXAMINATION OF TRAVEL FORECASTING TECHNIQUES USING THE GENERAL MODEL

So that conclusions can be drawn from the general model on the appropriateness of the policy-sensitive nature of travel forecasting models, the general model can be dismembered to resemble each of the travel models in turn. The assumptions involved in simplifying and altering the general model can be explicitly examined for their structural (causal) and statistical implications.

(Short-Run) Travel-Demand Models

The first simplification of the general model involves dropping back one step from the activity-location equations to the equations for travel. This requires the simplifying assumption that the distribution of activities in a region is given and fixed and that travel is modeled as a function of the fixed activities.

By omitting the equations for activity location in the general model, we are left with a partial equilibrium model, that is, a model that describes how part of the system behaves in order for it to be in equilibrium with the rest of the system. Thus, we model the behavior of the trip-maker who considers all trip-end opportunities and travel costs fixed. He chooses only his destination and method of travel because he has no control over the distribution of activities in the region or the (unit) transportation prices he must pay to obtain his desired outputs from those activities.

The first simplification of the general model returns us to the travel-demand formulation from which the final extension to the general model was made. The round-trip, two-travel-method model in product form appears as follows:

$$D^1_{iji} = a_1 p_{iji}^{1b_{11}} p_{iji}^{2c_{12}} A_{ki}^{d^i_{1k}} A_{kj}^{d^j_{1k}}$$

$$D^2_{iji} = a_2 p_{iji}^{2b_{22}} p_{iji}^{1c_{21}} A_{ki}^{d^i_{2k}} A_{kj}^{d^j_{2k}}$$

$$b \leq 0$$

$$c \geq 0$$

where

1,2 = method of travel 1 and 2 (such as mode, route, or time period);
D = round trips;
i = origin;
j = destination;
k = activity (output) type; and
p = vector of round-trip times and costs that must be expended on travel by method m between iji.

Or, the more general form,

$$D^m_{iji} = f\left(p_{iji}^{mb_{mm}},\ p_{iji}^{n\neq m\, c_{m,\, n\neq m}},\ A_{ki}^{d^i_{mk}},\ A_{kj}^{d^j_{mk}}\right)$$

where m = method of travel (1, . . ., m, n, . . ., M).

The (short-run) travel model states that trips by method m from origin i to destination j (or bundle of destinations j) and then back to i are some function of the activity systems at i and j and the price and service conditions by method m and all substitutable methods n. Trips by travel method are forecast directly in separate equations. Separate equations can be used to model the behavior of various socioeconomic groups. This is the basic model that has been estimated already by using urban travel data from Boston (4).

There are 2 principle consequences of separating the long-run demand or activity-location decision from the short-run travel decision that we can discuss on the basis of this simplification of the general model. The first relates to the logical and statistical problems introduced by omitting causal variables. The second relates to the separation itself.

The first consequence for short-run travel forecasting is that, because travel is a derived-demand quantity, equations such as those given above are incomplete. That is, travel should be modeled as an intermediate output of the larger urban system, as per the general model. The omission of important variables in a general model can cause inappropriate measurements of the effects of other variables (8). The statistical and operational consequences of this omission need thorough empirical and theoretical study.

The second consequence relates to the (modeling) separation itself of long- and short-run demand. In this regard, Lowry (9) notes:

> Since a stock is by definition the integral over time of the corresponding flow, it must also have the same determinants as the flow. [We note that travel by type to and from a point mirrors the amount and type of activity at that point, particularly when the activities are defined as travel-related outputs.] But if the model builder limits his attention to flows which occur over any short span of time, he can afford to take a number of shortcuts. Exogenous variables whose

> effects on stocks are visible only in the long run can be ignored or treated as fixed parameters.... By accepting the initial magnitude of a stock as historically "given," one avoids the necessity of replicating the past and can devote himself to modelling the events of the present and the near future.

However, by avoiding specific attention to the long-term effects contained in the general model, one also avoids structurally modeling those effects. Just as (structural) changes in network equilibrium are not modeled in the current conventional models, there may be and probably are structural long-term changes that are (of course) not modeled in a short-run travel-demand model.

However, this cannot lead us to conclude that the separation of long- and short-run forecasting is itself at fault. It leads us instead to the conclusion that the long-run models themselves must be structural. It also reminds us once again that our short-run models should incorporate relations among travel and its determinants that are expected to remain valid in the future.

Conclusions on the usefulness of short-run demand models in view of the possible consequences of separating short- and long-run models need much future research. This is a central problem in transportation systems analysis. The first problem, that of misspecification (in view of the lack thus far of a good short-run theory of travel demand), is more troublesome than the second. That is, separating long- and short-run travel-demand forecasting is itself not a problem, if a short-run model based on a plausible theory of short-run travel demand can be obtained, which is to say the two problem areas are really one. The general model appears to be a useful vehicle for further research into this question.

Meanwhile, in the absence of an estimated general model, short-run travel-demand models are the only travel-forecasting models available. These include the models presented and discussed in this section and the current conventional models used in the urban transportation studies during the 1960's. The remaining task, therefore, is to examine the general model further to help us evaluate the operationality in transportation planning of the current conventional short-run travel models.

Conventional Models

Trip Generation—Trip generation in the conventional model omits round trips and all costs of travel from the short-run travel model given above, leaving, for trip production,

$$q_i = "G_i" = f(A_{ki}) = d_1^i A_1 + d_2^i A_2 + \ldots$$

and for trip attraction,

$$q_i = "A_j" = f(A_{kj}) = d_1^j A_1 + d_2^j A_2 + \ldots$$

As noted earlier, the number of trips in the future is assumed in the conventional models to vary only as the activity levels vary. Nothing else influences the amount of travel, whether it be the price of travel (times and costs) by one travel method, the presence of substitute methods of travel, or the level of trip-end opportunities at the opposite trip end. Vis-à-vis the explicit lack of policy sensitivity of conventional trip-generation equations, the same conclusions as before may be drawn. However, collapsing the general model, or even the short-run travel-demand model given above, demonstrates how badly trip-generation equations are misspecified. That is, we can expect widely varying effects of the omitted variables to be attributed to the activity variables. The attribution by regression techniques of the effect of these omitted variables to the remaining variables can be expected to impair the accuracy of the effect of the activity variables on trip generation.

Trip Distribution—By dropping round trips and omitting all travel-method-specific equations but one, and by dropping out the terms for the substitutable travel methods from the one remaining (total) travel equation, the short-run travel model can be made

to look like the gravity model. That is, omitting terms from the product-form model yields

$$D_{ij} = ap_{ij}^{b} A_i A_j$$

Because the b coefficient is ≤ 0,

$$D_{ij} = \frac{aA_i A_j}{p_{ij}^{b}}$$

Replacing the activities by the G_i and A_j, obtained in trip generation, and solving in the usual manner for the constant a yield the functional form of the gravity model:

$$D_{ij} = \frac{G_i \dfrac{A_j}{p_{ij}^{b}}}{\sum_j \dfrac{A_j}{p_{ij}^{b}}}$$

Normal application of the gravity model omits consideration of the effects of substitute modes and omits a full set of travel times and costs expended in travel p. If an application did use some measure (or vector) of price on more than one mode, the proper signs of the b and c coefficients in each of their respective short-run travel-method-specific equations would have to be adhered to and the equations somehow added to preserve internal, albeit only partial, logic (11).

However, the important problem with conventional gravity-model trip distribution, however doctored as per the previous paragraph (and aside from its short-run nature), is again the problem of specification error. That is, the relations among travel methods and the activity variables are omitted from the equation. In their places are inserted fixed numbers of trips (generated and attracted) that must be adhered to (i.e., "balanced"). The travel-cost distribution (the purely descriptive, not causal trip-length frequency distribution) is also fixed. There is no chance to model the travel cost and trip-end benefit trade-off. In short, there is no calculation of a network equilibrium between cost of travel and benefit derived from engaging in various activities at trip destinations. The user is locked into 2 simple descriptions of the conditions that existed at the prevailing network equilibrium for the time and place the gravity model was calibrated.

Modal Split—Current post-distribution, modal-split models normally incorporate the largest set of price variables of any of the conventional models. However, the models operate on (split) the independently derived fixed trip distribution discussed above. Two major problems can be seen.

The first problem is the misspecification problem. That is, dropping out measures of activities at trip destinations causes inappropriate attribution of the effects of these variables on the included times and costs. The effects of the omitted destination variables, for example, can be expected to appear in the price variables in ways that serve to make (again) the partial effects of these variables inappropriate. For example, a simple case of this may cause the usual difficulty that modal-split models have in modeling CBD-oriented and non-CBD-oriented trips with the same model. This is because the influence of different price and service characteristics of trips can be expected to vary depending on the nature of the final goods and services consumed or employment obtained. Simple stratification of trips by trip purpose does not normally even ensure a good fit.

The second problem with conventional modal-split models arises from the lack of travel-method-specific (e.g., mode) travel prices. That is, travel times and costs are treated equally in modal-split models regardless of the travel mode on which they are incurred. This results in a unit change in travel time or cost having the same effect on relative usage of automobile or transit regardless of which is improved (or which is subject to increased congestion or travel cost).

Resources expended on travel should be modeled as mode (method) specific until it can be shown that easily measured resources (components of travel time and cost) can be treated independently of mode. There is evidence that they cannot. That is, elasticities of demand with respect to mode-specific times and costs differed substantially in the already estimated short-run travel-mode-specific demand models of the form given above (4).

It is quite likely that the models one works with for a long time constrain one from thinking freely in terms of how travelers make travel decisions. Modal split (for example) has no behavioral significance to the traveler. For evaluation (if desired), its arithmetic calculation may be made after mode-specific (method) travel-demand forecasts have been made.

Conclusion—The urban transportation studies of the 1950's and 1960's, using the current conventional models, were focused on providing information primarily on a single criterion of building to accommodate some fixed anticipated travel "demand." The studies were mainly content to publish long-range (as indeed required by section 134 of the 1962 Federal-Aid Highway Act) travel projections on the proposed transportation system as the primary justification for the recommended plan; that is, the facilities proposed were big enough to accommodate the anticipated traffic. Operationality in urban transportation planning today requires structural models that allow calculation of new network equilibrium levels of usage and congestion. This requires demand models that incorporate the individual traveler trade-offs caused by changing congestion levels inherent in facilities provided and not provided.

The examination in the previous section of the conventional travel models has been based on an explicit examination of some of the simplifying assumptions necessitated by dismemberment of the general model to look like them. The examination has shown something more alarming, however, than the shortcomings observed prior to the development of the general model in this paper. The alarming problem is that not only are the conventional models not policy sensitive (as concluded earlier) but also the specification errors (omitted variables, variable types, and whole equations) repeatedly raise the strong possibility of impaired accuracy of attribution and estimation of the effect of the policy variables that are included. Thus, misleading "policy" forecasts are possible (if not probable).

GENERAL CONCLUSIONS

Structural travel-demand models are required to implement operationality in transportation planning. Models are required that can be estimated with confidence that the effects attributed to policy variables are appropriately measured. Appropriate calculations are required of user and social costs reflecting true network equilibrium performance on the widely varying alternative transportation networks now being proposed in cities [and that may soon include innovative transportation alternatives as well (11)].

Travel-demand models must be based on a plausible and well-understood theory of travel behavior. The finely tuned descriptions of existing travel contained in the current conventional models have little relation to a plausible theory of travel and land use location. Of theory and method, more theory and less method are needed.

For practical reasons, also, short-run, policy-sensitive, travel-demand models of the type described earlier and already documented in the literature (3) are needed. Such models predict interzonal travel demand by travel method (e.g., mode) directly and employ relations only slightly more complex than those appearing in each of the separate conventional sequences of models. In ease of application, there appears to be little comparison. On the one hand, solutions are required of one equation directly for trips between a zonal pair (with resulting ease in disaggregating forecasts, a pressing need because of current concern with the distribution of costs and benefits as well as their aggregate values). On the other hand, manipulation of regional data files several times to achieve the same result is required. Errors are decreased with the new models, and introduction of the supply side is greatly facilitated. [This, of course, requires appropriate supply functions and appropriate interaction with the demand functions discussed here (3, 12).]

Further research is needed to evaluate whether in the long run other structural changes may make inappropriate our present separation of short-run travel forecasting from long-run land use forecasting. It may be that not incorporating certain long-run structural relations leads to inappropriate calculation of the long-run equilibrium between land use and travel. That is, not solving as one simultaneous set the relations between the demand for travel and the demand for goods and services output at the trip ends may lead to biased forecasts of either or both. However, the separation of long- and short-run demand seems now to be appropriate for at least practical reasons. Nevertheless, short-run travel forecasting with current conventional models appears inappropriate for both practical and structural reasons.

An approach to travel forecasting needed is one that reflects current societal values and not one that is grounded in past transportation planning values and practice. Rapid changes in values are taking place in our society. If policy-sensitive models are not rapidly implemented, travel forecasting stands in danger of being bypassed in transportation decision-making, and consideration of travel user benefits from transportation improvements will be bypassed with it.

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PRACTICAL IMPLICATIONS OF SOME FUNDAMENTAL PROPERTIES OF TRAVEL-DEMAND MODELS

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The 2 major approaches to finding equilibrium between supply and demand in a network are the indirect, which uses a sequential demand model, and the direct, which uses an explicit demand model. This paper describes the general share model that can be used in explicit form in a direct approach to computing equilibrium or in the form of a sequence of equations in an indirect approach. A number of practical implications follow from these theoretical results. A new model system should be developed without the serious limitations and internal inconsistencies of the urban transportation model system, which uses the indirect, sequential approach. Such a new system can be very general and designed to compute equilibrium with the GSM in a direct approach. Options can be provided with a rich variety of specific demand models, as special cases of the general share model, but within the same general structure. Efficient procedures for computing equilibrium may be developed by exploiting the elasticity properties of the general share model. As an immediate and practical step, present computer programs should be modified to compute a valid equilibrium, to include level of service explicitly and consistently at each step, especially trip generation, and to incorporate the special product models.

• THE OBJECTIVE of this paper is to present some recent results in the theory of transportation systems analysis. These results deal with properties of demand models. Although the results are largely theoretical, they have immediate practical application. The present paper stresses these practical implications and only summarizes the key aspects of the theoretical results; the theoretical material is presented more extensively elsewhere (1).

Several factors motivated the research that is summarized here:

1. The development of the theory of transportation systems analysis;
2. The need to have travel-demand models appropriate to the transportation issues with which we are now concerned, especially in urban transportation planning;
3. The significant weaknesses of the conventional approach to travel-demand forecasting used in urban transportation studies; and
4. The emergence of alternative approaches—direct demand models of an aggregate nature and behavioral models of a disaggregate nature.

THEORY

The theory of transportation systems analysis has emerged from several sources (2, 3, 4, 5). In outline, the problem of predicting the flows in a transportation system is a simple application of economic theory: The flows that will result from a particular transportation system T and the pattern of socioeconomic activities A can be determined by finding the resulting equilibrium in the transportation market. If V = volume of flow,

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L = level of service experienced by that volume, and F = (V,L) = flow pattern, then we find equilibrium by establishing a supply function S and a demand function D and by solving for the equilibrium flows F_o consistent with both relations (1):

$$\begin{bmatrix} L = S(V,T) \\ V = D(L,A) \end{bmatrix} \longrightarrow \begin{bmatrix} F_o = (V_o, L_o) \end{bmatrix} \qquad (1)$$

This is shown in Figure 1 with travel time t as the level of service L.

Although simple in outline, the application of the theory becomes complex in practice for several reasons:

1. The consumer considers many service attributes of the transport system when making a choice (e.g., line-haul travel time, transfer time, walk distance, out-of-pocket cost, and privacy), and thus L must be a vector with many components;
2. Determining the demand functions (as well as other elements) to use is difficult; and
3. The equilibrium occurs in a network, where flows from many origins to many different destinations interact and compete for the capacity of the network, and the form of these interactions is affected by the topology of the network.

Thus, fairly elaborate computational schemes are required to actually determine the equilibrium flows F_o for a particular (T,A).

In the case of a multimodal network, the symbol V represents an array of volumes

$$V = \{V_{klmp}\} \qquad (2)$$

for every k, l, m, and p, where V_{klmp} is the volume flowing from origin zone k to destination zone l via mode m and path p of that mode and where the braces indicate a set of elements V_{klmp}. Ideally, once we have established our demand and supply functions, we would then like to be able to turn directly to an equilibrium-calculating procedure to solve the 2 sets of relations to find the equilibrium flow pattern. The result of this computation would be the 2 arrays comprising that flow pattern:

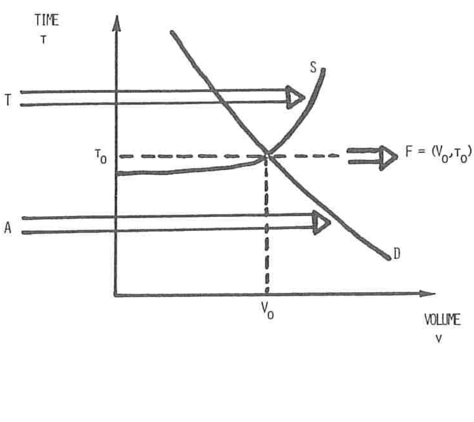
$$F_o = (V_o, L_o) \qquad (3)$$

where

$V_o = \{V_{klmp}\}$ for every k, l, m, and p; and
$L_o = \{L_{klmp}\}$ for every k, l, m, and p.

In words, we should get out of our equilibrium procedure the volumes, and the levels of service experienced by those volumes, from k to l by mode m and path p.

Figure 1. Basic theory.



Unfortunately, at this state of the science of transportation modeling, although several systems of transportation models exist, there is not even one operational model that solves for these equilibrium flows exactly and directly. Each of the available systems of models represents a different operational approach to computing equilibrium in transport networks; the differences in these approaches are reflected both in the computational algorithms and in the structure of the demand models that are used. None of these produces an exact equilibrium.

Alternative Approaches

We deal here with only one of Wardrop's "principles"; we will not discuss the other,

concerned with global optimization of the flow pattern, as this is inapplicable to urban transportation flow prediction.

One particular computational scheme is that used in urban transportation planning studies. In this indirect approach, the equilibrium flows are estimated in a sequence of steps, commonly called trip generation, trip distribution, modal split, and traffic assignment (9, 10).

Correspondingly, the demand function D is represented as a sequence of functions: trip generation (and attraction) equations, trip distribution procedures (including the friction factor transformation of L), modal-split equations, and minimum-path rules of the traffic assignment procedures. We will refer to this approach as the urban transportation model system (UTMS).

More recently, alternative approaches have been developed. One approach uses explicit demand models to estimate the equilibrium flows in a direct approach, i.e., in a single step instead of in a sequence of steps as in the UTMS approach. Thus, such explicit demand models combine the functions of generation, distribution, and modal split (and, potentially, route choice) into a single process. The first such models were developed for forecasting intercity passenger travel for the Northeast Corridor Project of the U. S. Department of Transportation. They began with the Kraft-SARC model and were followed by the work of McLynn and others (3, 11, 12, 13, 14). Later work extended these models to urban travel (15, 16). These types of explicit demand models were first used for transportation network analysis in the simulation studies for the Northeast Corridor Project. In the DODOTRANS system of computer models, a number of these models are available for use in computing equilibrium flows in networks (17).

Other major directions of work include the disaggregate approach of the so-called behavioral models (18, 19), the development of aggregate models from entropy considerations (20), and the Harvard model system for national transportation planning (21).

Careful appraisal of all of these approaches indicates that, viewed simply as a computational problem, the task of computing the equilibrium of supply and demand in a network remains a difficult one. The UTMS approach has significant limitations, as we shall see shortly; the Northeast Corridor and Harvard models make special assumptions that essentially make their approaches not generally applicable. The DODOTRANS system is the most theoretically acceptable of the currently operational and practical approaches; however, the convergence and uniqueness properties of its computational procedures are not wholly satisfactory either. Several promising approaches are under development (22, 23, 24).

Thus, although the theory of what should be done is clear, at present there are a number of alternative approaches that are or can be taken to predict flows in networks as the equilibrium of supply and demand. Each approach involves specific assumptions, both explicit and implicit, in the choice of demand models and of the computational procedures for determining equilibrium. Many alternative assumptions and computational procedures are possible. Very serious biases may occur in the computed flow patterns, as compared with the "true" equilibrium, if the assumptions and computational approaches are not carefully considered. Therefore, it is essential that the transportation analyst have a sound understanding of the role of demand models in the equilibrium calculation so that he can appraise possible biases.

APPRAISAL OF THE URBAN TRANSPORTATION MODEL SYSTEM

The UTMS is the most widely used transportation systems analysis approach. It has been applied in more than 200 cities in the United States and in many other cities around the world. The development and the institutionalization of this approach during the past 15 years are major accomplishments; it is the first large-scale application of modern systems analysis techniques to problems of the civil sector.

In the UTMS, the travel-demand models—and the equilibrium computations—are structured into a sequence of 4 major steps: trip generation, trip distribution, modal split, and traffic assignment. Essentially, this amounts to estimating V_{klmp} in a series of successive approximations: first V_k, then V_{kl}, then V_{klm}, and finally V_{klmp}.

It is useful to examine the UTMS critically from the perspective of equilibrium theory and the challenge of today's urban transportation problems (1). It seems obvious that the following conditions should be met by any set of demand models and equilibrium-calculating procedures.

1. Level of service L should enter into every step, including trip generation (unless an analysis of the data indicates in a specific situation that trip generation is, in fact, independent of level of service for all market segments over the full range of levels of service to be studied).
2. The level of service attributes used should be as complete as necessary to adequately predict traveler behavior. For example, time reliability, number of transfers, and privacy should be included if empirical evidence indicates these are important.
3. The same attributes of service level should influence each step (unless the data indicate otherwise). For example, transit fares, automobile parking charges, walking distances, and service frequencies should influence not only modal split but also assignment, generation, and distribution.
4. The process should calculate a valid equilibrium of supply and demand; the same values of each of the level-of-service variables should influence each step. For example, the travel times that are inputs for modal split, distribution, and even generation should be the same as those that are outputs of the assignment. If necessary, iteration from assignment back to generation, distribution, and so on should be done to get this equilibrium.
5. The levels of service of every mode should influence demand. Congestion, limited capacity (e.g., parking lots), and fares of each mode should (in general) affect not only its own demand but also the demand for other modes at all steps (generation, distribution, modal split, and assignment). That is, there should be provision for explicit cross elasticities.
6. The several demand functions for each step should be internally consistent (in the sense defined in the next section).
7. The estimation procedures should be statistically valid and reproducible.

Careful examination of the UTMS indicates that it violates each of these conditions (1). As a consequence, serious questions can be raised about the biases and limitations of the flow predictions resulting from use of the UTMS. Although the UTMS does have important advantages, these do not outweigh its very serious liabilities.

What is desirable is an improved approach that overcomes these limitations by meeting the conditions listed. The results described here do this and, at the same time, preserve the advantages of the indirect approach to equilibrium used in the UTMS.

BASIC DEFINITIONS

Notation

The following notation is used in this paper:

V_{klmp} = volume of trips going from zone k to zone l by mode m and path p;
V_{klm} = volume of trips going from zone k to zone l by mode m;
V_{kl} = volume of trips going from zone k to zone l;
V_k = volume of trips originating in zone k;
V_T = volume of trips (interzonal) in the region;
$\underline{A}$ = vector of variables describing the socioeconomic activity system;
$\underline{a}$ = vector of parameters applying to $\underline{A}$;
$\underline{X}_{klmp}$ = vector of S level-of-service variables ($i = 1, 2, \ldots, S$) describing the transportation system characteristics as experienced by trips going from zone k to zone l by mode m and path p;
$\underline{X}$ = $\{\underline{X}_{klmp}\}$ = set of all level-of-service characteristics for all paths p of all modes m between all origins k and all destinations l;
$\underline{w}$ = vector of parameters applying to $\underline{X}$;
R_{klmp} = $f(\underline{X},\underline{w})$ = combined effect of all level-of-service characteristics of all modes as they influence trips going from zone k to zone l by mode m and path p;

$R_{klmq,p}$ = combined effect of all level-of-service characteristics of mode q as they influence trips going from zone k to zone l by mode m and path p;
Z = $f(\underline{A},\underline{a})$ = combined effect of all activity-system characteristics;
Y = $f(\underline{Z},\underline{R})$ = combined effect of all activity-system and level-of-service characteristics; and
ψ = demand function.

V_{klmp}, V_{klm}, V_{kl}, and V_T are in general not the same as the various partial sums obtained by aggregating over all values of one or more subscripts. For example, we will write $V_{klm.}$ and $V_{kl..}$ when we do mean

$$\left.\begin{aligned} V_{klm.} &= \sum_p V_{klmp} \\ V_{kl..} &= \sum_m V_{klm.} \end{aligned}\right\} \quad (4)$$

and so on. In general,

$$\left.\begin{aligned} V_{klm.} &\neq V_{klm} \\ V_{kl..} &\neq V_{kl} \end{aligned}\right\} \quad (5)$$

except where otherwise indicated.

The definition of $R_{klmq,p}$ represents a cross-elasticity type of effect where the service attributes of mode q affect the demand for path p of mode m.

Those definitions are illustrated as follows (we assume each mode m has only 1 path, and therefore we drop the subscript p):

$$V_{klm} = a_1 P_k^{a_2} E_l^{a_3} t_{klm}^{a_4} c_{klm}^{a_5} \quad (6)$$

In this simple product form of demand model, we have

$\underline{A}$ = (P_k, E_l) = population at origin k, employment at destination l;
$\underline{X}$ = (t_{klm}, c_{klm}) = time, cost;
$\underline{a}$ = (a_1, a_2, a_3);
$\underline{w}$ = (a_4, a_5);
Z_{kl} = $a_1 P_k^{a_2} E_l^{a_3}$;
R_{klm} = $t_{klm}^{a_4} c_{klm}^{a_5}$;
Y_{klm} = $Z_{kl} \cdot R_{klm}$; and
V_{klm} = $\psi_{klm}(Y) = Z_{kl} \cdot R_{klm}$.

Types of Models

The general demand model system is

$$V = \psi(V,Y) \quad (7)$$

This is implicit in that V appears on the right side as well as on the left of the equation. Thus, even if we know all elements of Y, including the level of service $\underline{X}$, we would still generally have difficulty computing the value(s) of V to satisfy the equation (for example, the iteration of the modified gravity model to balance productions and attractions in the UTMS).

The implicit system may be several equations:

$$\left.\begin{aligned} V_1 &= \psi_1(V,Y) \\ V_2 &= \psi_2(V,Y) \\ &\cdots \\ V_N &= \psi_N(V,Y) \end{aligned}\right\} \quad (8)$$

or it may be a single equation:

$$V_{klmp} = \psi(V_{klmp}, Y) \tag{9}$$

One special case of the general implicit system is the explicit system of demand models:

$$V_{klmp} = \psi(Y) \tag{10}$$

in which V appears only on the left side of the equation. The McLynn, Kraft-SARC, and similar models are explicit models.

Another special case of the general implicit system is the sequential implicit:

$$\left.\begin{aligned} V_T &= \sigma_1(Y) \\ V_k &= \sigma_2(V_T;Y) \\ V_{kl} &= \sigma_3(V_k;Y) \\ V_{klm} &= \sigma_4(V_{kl};Y) \\ V_{klmp} &= \sigma_5(V_{klm};Y) \end{aligned}\right\} \tag{11}$$

In principle, the sequential form can be transformed into an explicit form:

$$V_{klmp} = f(Y) \tag{12}$$

where $f = f(\sigma_1, \sigma_2, \sigma_3, \sigma_4, \sigma_5)$.

The explicit form can be used in the direct approach to computing equilibrium. The sequential form requires an indirect approach in a series of steps; when transformed into an explicit form as in Eq. 12, the direct approach can be used.

The forecasting method used in urban transportation studies, the UTMS, is based on a sequential implicit system of demand models used in an indirect approach to equilibrium: σ_1 and σ_2 correspond to the trip generation equations, σ_3 to trip distribution, σ_4 to modal split, and σ_5 to traffic assignment.

Consistency and Beta Conditions

The question that naturally arises is, Under what conditions will both direct and indirect approaches give the same results? Although the question cannot be answered in general, one necessary (but not sufficient) condition can be identified: The sequential implicit system must be internally consistent, defined as follows.

We would like the set of volumes V produced by a sequential implicit system (Eq. 11) to meet this obvious condition:

$$\left.\begin{aligned} V_T &= \sum_k V_k \\ V_k &= \sum_l V_{kl} \\ V_{kl} &= \sum_m V_{klm} \\ V_{klm} &= \sum_p V_{klmp} \end{aligned}\right\} \tag{13}$$

for all Y.

If a sequential implicit system $V = \psi(V,Y)$ produces volumes V that meet the conditions in Eq. 13 for all Y, we say it is internally consistent.

By Eq. 11, this leads immediately to the following necessary and sufficient beta conditions for a sequential implicit system to be internally consistent:

$$\left.\begin{aligned}
\sigma_1(Y) &= \sum_k \sigma_2(V_T, Y) \\
\sigma_2(V_T, Y) &= \sum_l \sigma_3(V_k, Y) \\
\sigma_3(V_k, Y) &= \sum_m \sigma_4(V_{kl}, Y) \\
\sigma_4(V_{kl}, Y) &= \sum_p \sigma_5(V_{klm}, Y)
\end{aligned}\right\} \qquad (14)$$

These conditions seem reasonable by themselves. They are also necessary conditions if the explicit and sequential implicit forms of a model are to give the same equilibrium flow-pattern predictions.

GENERAL SHARE MODEL

The general share model (GSM) is defined as

$$V_{klmp} = \alpha(Y) \cdot \beta_k(Y) \cdot \gamma_{kl}(Y) \cdot \delta_{klm}(Y) \cdot \omega_{klmp}(Y) \qquad (15)$$

where $Y = f(R,Z) = f(\underline{A}, \underline{a}, \underline{X}, \underline{w})$ as in the notation defined earlier, and α, β, γ, δ, ω are functions that meet the following range conditions for all values of Y:

$$\left.\begin{aligned}
&0 \le \alpha(Y) \\
&0 \le \beta_k \le 1, \sum_k \beta_k(Y) = 1 \\
&0 \le \gamma_{kl}(Y) \le 1, \sum_l \gamma_{kl}(Y) = 1 \text{ for every } k \\
&0 \le \delta_{klm}(Y) \le 1, \sum_m \delta_{klm}(Y) = 1 \text{ for every } k, l \\
&0 \le \omega_{klmp}(Y) \le 1, \sum_p \omega_{klmp}(Y) = 1 \text{ for every } k, l, m
\end{aligned}\right\} \qquad (16)$$

The name "share model" is used because each of the terms α, β, γ, δ, and ω "splits" the flow volume into successive "shares." (These relations can be derived by summing Eq. 15 over the various subscripts p, m, l, and then k and using Eq. 16.) Hence,

1. The total level of travel in the region is given by $\alpha(Y)$:

$$V_T = \alpha(Y) \qquad (17)$$

2. Of this total travel, the fraction that originates in zone k is given by $\beta_k(Y)$:

$$V_k = \beta_k \cdot V_T \qquad (18)$$

3. The fraction that originates in zone k and has zone l as its destination is given by $\gamma_{kl}(Y)$:

$$V_{kl} = \gamma_{kl} \cdot V_k \qquad (19)$$

4. The fraction that goes from zone k to zone l and uses mode m (the modal split) is given by $\delta_{klm}(Y)$:

$$V_{klm} = \delta_{klm} \cdot V_{kl} \qquad (20)$$

5. Finally, the fraction that goes from zone k to zone l and uses path p of mode m is given by $\omega_{klmp}(Y)$:

$$V_{klmp} = \omega_{klmp} \cdot V_{klm} \tag{21}$$

These relations can be derived from the basic definition of the GSM.

Thus, although the GSM is itself explicit, we can also write the GSM (Eq. 15) in an alternative sequential implicit form:

$$\left.\begin{aligned} V_T &= \alpha(Y) \\ V_k &= \beta_k(Y) \cdot V_T(Y) \\ V_{kl} &= \gamma_{kl}(Y) \cdot V_k(Y) \\ V_{klm} &= \delta_{klm}(Y) \cdot V_{kl}(Y) \\ V_{klmp} &= \omega_{klmp}(Y) \cdot V_{klm}(Y) \end{aligned}\right\} \tag{22}$$

with conditions (Eq. 16) as before.

This is analogous to the sequential implicit demand model used in the UTMS indirect approach to equilibrium, as follows:

$$\left.\begin{aligned} V_k &= \beta_k \cdot \alpha \\ V_{kl} &= \gamma_{kl} \cdot V_k \\ V_{klm} &= \delta_{klm} \cdot V_{kl} \\ V_{klmp} &= \omega_{klmp} \cdot V_{klm} \end{aligned}\right\} \tag{23}$$

for trip generation, trip distribution, modal split, and traffic assignment respectively. In a later section, we show how this analogy enables us to overcome the biases and limitations of the UTMS as currently implemented.

PROPERTIES OF THE GENERAL SHARE MODEL

The following important properties of the GSM can be demonstrated. Details are given in another paper (1).

Theorem 1: The GSM can be expressed in both explicit and sequential implicit forms; the sequential implicit form is internally consistent.

Theorem 2: Any explicit demand model system can be expressed as a GSM.

Theorem 3: Any internally consistent, sequential implicit demand system can be expressed as a GSM.

Theorem 4: For every explicit demand system, there is a corresponding internally consistent sequential implicit system, and conversely.

The proof of theorem 1 follows from Eqs. 15, 22, and 16 and the beta conditions (Eq. 14).

The proof of theorem 2, although simple, is interesting because it offers a constructive method for getting the GSM for an explicit system:

1. Begin with the explicit model,

$$V_{klmp} = f(Y) \tag{24}$$

2. Derive the various partial sums,

$$V_{klm.}, V_{kl..}, V_{k...}, V_{....} \tag{25}$$

by

$$V_{klm.} = \sum_p V_{klmp} \text{ and so on} \tag{26}$$

3. Derive the "split fractions,"

$$\left.\begin{aligned}
\omega_{klmp} &= \frac{V_{klmp}(Y)}{V_{klm.}(Y)} \\
\delta_{klm} &= \frac{V_{klm.}(Y)}{V_{kl..}(Y)} \\
\gamma_{kl} &= \frac{V_{kl..}(Y)}{V_{k...}(Y)} \\
\beta_k &= \frac{V_{k...}(Y)}{V_{....}(Y)} \\
\alpha &= V_{....}(Y)
\end{aligned}\right\} \quad (27)$$

4. Construct the GSM as the product of these split fractions,

$$V_{klmp} = \alpha \cdot \beta_k \cdot \gamma_{kl} \cdot \delta_{klm} \cdot \omega_{klmp} \quad (28)$$

In particular, all of the existing explicit demand models, such as the Kraft-SARC, McLynn, and Baumol-Quandt, are special cases of the GSM.

The proof of theorem 3 is also constructive and follows from the definition of a sequential implicit model (Eq. 11):

$$\left.\begin{aligned}
\alpha &\equiv V_T = \sigma_1(Y) \\
\beta_k &\equiv V_k/V_T = \frac{\sigma_2(V_T, Y)}{\sigma_1(Y)} \\
\gamma_{kl} &\equiv V_{kl}/V_k = \frac{\sigma_3(V_k, Y)}{\sigma_2(V_T, Y)} \\
\delta_{klm} &\equiv V_{klm}/V_{kl} = \frac{\sigma_4(V_{kl}, Y)}{\sigma_3(V_k, Y)} \\
\omega_{klmp} &\equiv V_{klmp}/V_{klm} = \frac{\sigma_5(V_{klm}, Y)}{\sigma_4(V_{kl}, Y)}
\end{aligned}\right\} \quad (29)$$

Because the sequential model is specified as internally consistent, we know that the beta conditions (Eq. 14) will apply, and, therefore, Eq. 29 will meet the range conditions on the GSM (Eq. 16).

The proof of theorem 4 follows from the preceding theorems. Given any explicit system, by theorem 2 we can get a corresponding GSM. By theorem 1, this GSM has a corresponding sequential implicit form that is internally consistent, completing the first part of the proof. To show the converse, given any internally consistent sequential implicit system, by theorem 3 we find a corresponding GSM, and then by theorem 1 we have a corresponding explicit form.

These results, especially theorem 4, have very important practical implications, as described in the next section.

Additional properties of the GSM are described elsewhere (1). To summarize: The elasticities of the GSM and its components take a particularly useful form, which suggests directions for development of efficient equilibrium algorithms. The form of the GSM can be given a probabilistic interpretation, based on the range conditions (Eq. 16); this can lead to an explicit bridge between disaggregate stochastic models and aggregate models. The travel behavior of different market segments can be explicitly represented in different special cases of the GSM.

IMPLICATIONS

Because of space limitations, only a few of the major implications of the theoretical results can be presented here.

Families of Demand Models

The theorems given in the preceding section indicate the very general nature of the GSM. It is particularly interesting to explore various families of specific demand models that arise as special cases of the GSM and to see the relation of existing models to these.

In this discussion, for simplicity, we will ignore the complications introduced by assignment of flows to paths (i.e., network assignment) by assuming only 1 path p of each mode m and dropping the subscript p. This assumption can easily be relaxed.

Recall that α, β, γ, and δ are all functions of $Y = f(Z,R)$, where $Z = f(\underline{A},\underline{a})$ and $R = f(\underline{X},\underline{w})$. These are very general and provide for a great deal of flexibility in designing specific demand models.

For example, consider first the level of service (we drop the subscripts k, l, and m for the moment) $L = \underline{X} = (x_1, x_2, \ldots, x_i, \ldots, x_s)$ where perhaps we have the following specific level-of-service variables: x_1 = travel time, in-vehicle portion; x_2 = travel time, out-of-vehicle portion; x_3 = out-of-pocket cost; x_4 = frequency of service; and so on.

Alternative forms of R are as follows:

1. $R = w_0 + w_1x_1 + w_2x_2 + w_3x_3$;
2. $R = w_0e^U$, where $U = w_1x_1 + w_2x_2 + w_3x_3$;
3. $R = f(x_1)$;
4. $R = w_0x_1^{w_1}x_2^{w_2}x_3^{w_3}x_4^{w_4}$;
5. $R_m = \dfrac{w_{m0}\, x_{m1}^{w_1} x_{m2}^{w_2}}{\sum\limits_q w_{q0}\, x_{q1}^{w_1} x_{q2}^{w_2}}$; and
6. $R_m = w_{m0}\,(x_{m1}^{w_{m1}} x_{m2}^{w_{m2}}) \prod\limits_{q \neq m} R_{mq}$, where $R_{mq} = x_{q1}^{w_{mq1}} x_{q2}^{w_{mq2}}$.

Form 1 shows a value-of-time formulation, where w_2/w_1 expresses the relative values placed on in-vehicle and out-of-vehicle times, and w_1/w_3 expresses the value-of-time equivalency in cents per minute. Form 2 shows an exponential transform of a linear cost as used in the Twin Cities modal-split model (25). In form 3, f corresponds to a friction-factor transformation of travel time as used in typical gravity model applications. Form 4 is a general product form, and form 5 is that used in the McLynn model. Form 6 is the form used in the Kraft-SARC model, where w_{mi} is a direct elasticity, and w_{mqi} is a cross elasticity, reflecting the effect of mode q's level of service on travel by mode m.

The generality of the GSM should be clear from these examples and from theorem 4. As shown in Figure 2, the GSM includes as special cases all of the explicit demand models developed to date—for example, the Baumol-Quandt, McLynn, and Kraft-SARC are all special cases of the general direct demand model (GDDM), which is in turn a special case of the GSM. Further, another sequence of models can be formulated: the special product models (SPM), of which a modified form of the UTMS is a special case.

UTMS Models: Special Product Models

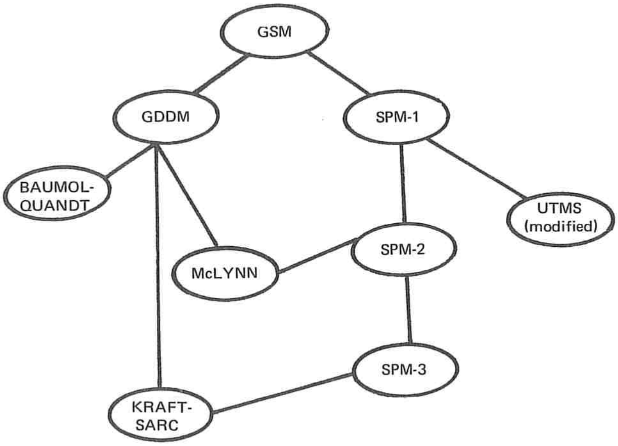
We begin with a very general form, the special product model 1 (SPM-1), defined as

$$V_{klm} = \left(\frac{R_{klm}}{R_{kl.}}\right)\left(\frac{Z_l \cdot R_{kl.}^{\delta_{10}}}{\sum\limits_l Z_l \cdot R_{kl.}^{\delta_{10}}}\right)\left[\frac{Z_k\left(\sum\limits_l Z_l R_{kl.}^{\delta_{11}}\right)^{\delta_{20}}}{\sum\limits_k Z_k\left(\sum\limits_l Z_l R_{kl.}^{\delta_{11}}\right)^{\delta_{20}}}\right]\left\{\left[\sum_k Z_k\left(\sum_l Z_l R_{kl.}^{\delta_{12}}\right)^{\delta_{21}}\right]^{\delta_{30}}\right\} \tag{30}$$

In sequential form, SPM-1 is as follows: trip generation, total,

$$V_T = \left[\sum_k Z_k\left(\sum_l Z_l R_{kl.}^{\delta_{12}}\right)^{\delta_{21}}\right]^{\delta_{30}} \tag{31}$$

Figure 2. Families of models.



trip generation, zonal,

$$V_k = V_T \left[\frac{Z_k \left(\sum_l Z_l R_{kl.}^{\delta_{11}} \right)^{\delta_{20}}}{\sum_k Z_k \left(\sum_l Z_l R_{kl.}^{\delta_{11}} \right)^{\delta_{20}}} \right] \tag{32}$$

trip distribution,

$$V_{kl} = V_k \left(\frac{Z_l R_{kl.}^{\delta_{10}}}{\sum_l Z_l R_{kl.}^{\delta_{10}}} \right) \tag{33}$$

and modal split,

$$V_{klm} = V_{kl} \left(\frac{R_{klm}}{R_{kl.}} \right) \tag{34}$$

If there is more than 1 path in each mode, traffic assignment can be added,

$$V_{klmp} = \left(\frac{R_{klmp}}{R_{klm.}} \right) V_{klm}$$

and corresponding changes made in Eq. 30.

From Eq. 34, the resistance R_{klm} should be a positive conductance measure; e.g., $R_{klm} = t_{klm}^{-a}$. Thus, the signs of δ_{10}, δ_{11}, and δ_{12} should be positive also.

To see the relation to the UTMS, we can examine a special case of SPM-1, obtained by setting the parameters as follows:

$$\delta_{10} = \delta_{11} = \delta_{12} \equiv \delta_1 \tag{35}$$

$$\delta_{20} = \delta_{21} \equiv \delta_2 \tag{36}$$

$$\delta_{30} \equiv \delta_3 \tag{37}$$

This leads to special product model 2 (SPM-2):

$$V_{klm} = \left(\frac{R_{klm}}{R_{kl.}}\right)\left(\frac{Z_1R_{kl.}^{\delta_1}}{\sum\limits_{1} Z_1R_{kl.}^{\delta_1}}\right)\left[\frac{Z_k\left(\sum\limits_{1} Z_1R_{kl.}^{\delta_1}\right)^{\delta_2}}{\sum\limits_{k} Z_k\left(\sum\limits_{1} Z_1R_{kl.}^{\delta_1}\right)^{\delta_2}}\right]\left\{\left[\sum_{k} Z_k\left(\sum_{1} Z_1R_{kl.}^{\delta_1}\right)^{\delta_2}\right]^{\delta_3}\right\} \tag{38}$$

SPM-2 can also be written as follows:

$$V_{klm} = R_{klm} \cdot Z_k \cdot Z_1 \left\{ R_{kl.}^{\delta_1 - 1} \left(\sum_{1} Z_1R_{kl.}^{\delta_1}\right)^{\delta_2 - 1} \left[\sum_{k} Z_k\left(\sum_{1} Z_1R_{kl.}^{\delta_1}\right)^{\delta_2}\right]^{\delta_3 - 1}\right\} \tag{39}$$

Now we introduce one further set of special conditions:

$$\delta_1 = \delta_2 = \delta_3 \equiv 1 \tag{40}$$

which leads to special product model 3 (SPM-3):

$$V_{klm} = R_{klm} \cdot Z_k \cdot Z_1 \tag{41}$$

To see the relationship with the standard UTMS, we consider the sequential form of SPM-2 as follows: trip generation, total,

$$V_T = \left[\sum_{k} Z_k\left(\sum_{1} Z_1R_{kl.}^{\delta_1}\right)^{\delta_2}\right]^{\delta_3} \tag{42}$$

trip generation, zonal,

$$V_k = V_T \left[\frac{Z_k\left(\sum\limits_{1} Z_1R_{kl.}^{\delta_1}\right)^{\delta_2}}{\sum\limits_{k} Z_k\left(\sum\limits_{1} Z_1R_{kl.}^{\delta_1}\right)^{\delta_2}}\right] \tag{43}$$

trip distribution,

$$V_{kl} = V_k \left(\frac{Z_1R_{kl.}^{\delta_1}}{\sum\limits_{1} Z_1R_{kl.}^{\delta_1}}\right) \tag{44}$$

and modal split,

$$V_{klm} = V_{kl}\left(\frac{R_{klm}}{R_{kl.}}\right) \tag{45}$$

In its first form, the classical gravity model of early transportation modeling was simply

$$V_{kl} = Z_k \cdot Z_1 \cdot R_{kl} \tag{46}$$

where Z_k was the population at $k(B_k)$; Z_1 was the population or employment at $1(B_1)$; and R_{kl} was distance or travel time to the power θ, where θ was approximately -2:

$$V_{kl} = \frac{B_k \cdot B_1}{t_{kl}^{-\theta}} \tag{47}$$

SPM-3 (Eq. 41) is simply a generalized gravity model. In later forms, the gravity model was normalized in this way:

$$V_{kl} = \left(\frac{Z_l \cdot R_{kl}}{\sum\limits_{l} Z_l \cdot R_{kl}} \right) Z_k$$

$$= B_k \left(\frac{B_l t_{kl}^{\theta}}{\sum\limits_{l} B_l t_{kl}^{\theta}} \right) \tag{48}$$

In this form, the term

$$P_{kl} = \frac{B_l}{t_{kl}^{-\theta}} \tag{49}$$

is referred to as the "potential" of zone k. The term

$$P_{k.} = \sum_{l} \left(\frac{B_l}{t_{kl}^{-\theta}} \right) \tag{50}$$

is called the "accessibility," reflecting a weighted average of the attractiveness of the various destinations as measured by B_l, weighted by the difficulty of access to those destinations measured by $t_{kl}^{-\theta}$.

Now, to be more general, we can define a generalized potential as

$$P_{kl} \equiv Z_l \cdot R_{kl.}^{\delta_1} \tag{51}$$

and a generalized accessibility as

$$P_{k.} = \sum_{l} Z_l R_{kl.}^{\delta_1} \tag{52}$$

where we allow a number of variables to enter the Z and R terms; and in particular, we have cross-elasticity terms $R_{klmq,p}$ in R (as in form 6 given in an earlier section).

Thus, the distribution stage of SPM-2 (Eq. 44) is a generalized gravity model, with population B_k replaced by the more general measure of the intensity of the activity system $Z = f(\underline{A}_k, \underline{a})$ and time t_{kl}^{θ} replaced by a more general measure of the resistance of the transportation system R_{kl}.

Now let us examine the generation stage. We substitute P_{kl} and $P_{k.}$ and get

$$V_T = \left(\sum_{k} Z_k \cdot P_{k.}^{\delta_2} \right)^{\delta_3} \tag{53}$$

$$V_k = V_T \left(\frac{Z_k \cdot P_{k.}^{\delta_2}}{\sum\limits_{k} Z_k \cdot P_{k.}^{\delta_2}} \right) \tag{54}$$

Thus, to get total trips in the region, we calculate a trip-generating potential,

$$G_k \equiv Z_k \cdot P_k^{\delta_2} \tag{55}$$

This potential reflects both the level of the activity system at k and Z_k and the influence of $P_{k.}$, the accessibility of k (an average over all destinations). The exponent δ_2 scales the effect of accessibility on trip generation (and can be zero when appropriate). The total trips generated in the region V_T depends on $G.$, the sum of these trip-generating potentials G_k for all zones; the exponent δ_3 establishes the extent to which V_T is sensitive to G. The trips generated in any zone k are proportional to its share of this sum of potentials:

$$V_k = V_T \left(\frac{G_k}{G.}\right), \quad V_T = G.^{\delta_3} \tag{56}$$

In a sense, then, the generation stage of SPM-2 is also a generalization of the classical gravity model. However, instead of assuming that V_k is a constant, independent of level of service, as in the UTMS, we are establishing V_k as a function of level of service consistent with the gravity-model-like approach to distribution.

Thus, as this sequential form shows, SPM-2 (Eq. 38) is a more general version of the UTMS, but with the following properties:

1. Any desired level-of-service variables, with any desired direct and cross elasticities, can be incorporated in the resistance R_{klm} used to characterize a particular mode.
2. The same resistance term enters every step of the process—generation and distribution as well as modal split—in a consistent way, either as R_{klm} or as part of the sum $R_{kl..}$
3. To compute a valid equilibrium, we can use the explicit form as in Eq. 38 to accomplish the calculations in one step, using a direct approach. If there should be some reason why we want to compute equilibrium in an indirect manner, we can use the sequential form (Eqs. 42, 43, 44, and 45), which is internally consistent.
4. The various parameters δ establish the influence of level of service on various aspects of travel behavior and may have different values to reflect the behavior of different segments of the urban travel market (1). (SPM-1 is even more general and provides even more flexibility for the analyst in developing alternative special cases for different conditions.)
5. The model can be estimated by using standard statistical techniques (26). Therefore, we have met all of the conditions outlined earlier in this paper.

CONCLUSIONS AND RECOMMENDATIONS

Although the results presented here are relatively theoretical and abstract, they have practical implications. These implications are summarized in the following sections in the form of recommendations.

Summary of the Theoretical Results

Before specific recommendations are described, it will be useful to summarize the major features of the theoretical results that have been presented and their implications:

1. A desirable property of a sequential implicit system is that it be internally consistent (Eq. 13) for which the beta conditions (Eq. 14) are necessary and sufficient.
2. The GSM (Eq. 15) has both explicit and sequential implicit forms; the sequential implicit form is internally consistent. Equilibrium can be computed in the direct approach with the explicit form or in the indirect approach with the sequential implicit form.
3. Any explicit demand model can be expressed as a GSM. For example, the Northeast Corridor and the urban models (Kraft-Domencich-Vallette and Plourde) are all special cases of the GSM.
4. Any internally consistent, sequential implicit demand system can be expressed as a GSM.
5. For every explicit demand model, there is a corresponding internally consistent sequential implicit form, and conversely.
6. As a consequence,

a. We are completely free to choose whether to compute equilibrium via the direct or indirect approaches (if direct, we use the explicit form, or, if indirect, the sequential implicit form);

b. The UTMS is a sequential implicit system, can be modified to be internally consistent, and in modified form can be expressed as a GSM (therefore, the shortcomings

of the indirect approach to computing equilibrium can be overcome by using the explicit form of the modified UTMS in a direct approach to computing equilibrium); and

c. Any explicit demand model can be expressed in its corresponding sequential implicit form and used in an indirect approach to equilibrium (thus, the analyst can have the capability, if desired, to control each step of a travel prediction with an explicit model in the same way each step of the UTMS can be controlled).

7. The GSM suggests a variety of possible demand models. In particular, a series of special product models (SPM-1, SPM-2, and SPM-3) provide a family of model forms (Eqs. 30 to 45), which are similar to the UTMS, the classical gravity model, and explicit models such as the Kraft-SARC and the McLynn, but provide a rich variety of options. These options can be used to represent the travel behavior patterns of different market segments. (Another attractive feature of the GSM arises from the properties of its elasticities. These suggest the development of efficient approximation techniques to explore the effects on flow volumes of small changes in the transportation system.)

Attitudinal Change

The UTMS was a major accomplishment for its time. The profession and the governmental transportation agencies (federal, state, and local) must recognize that the UTMS is no longer satisfactory; it is neither relevant to the practical issues that must be addressed in the urban transportation studies of today (1, 27) nor acceptable when viewed from a theoretical perspective. The UTMS should be neither completely discarded nor allowed to remain unchanged as the basic working tool of urban transportation analysis.

A new generation of transportation analysis tools is required. Development of new systems should build on the several directions of current research, as well as the practical experience gained from the UTMS. The recommendation is that we begin by asking not whether but how.

A Direction

We cannot here lay out the preliminary design of a new generation of urban transportation models. However, the theoretical results presented in earlier sections suggest one possible approach. Recommendations are as follows:

1. A model system should treat the transportation system of a region as a single multimodal system, taking each trip from door to door through any possible mix of transport facilities.
2. A model system should allow explicit treatment of any number of market segments and should allow each market segment to have different behavior patterns. These differences may be expressed not only in the values of the parameters of the demand functions (e.g., the values of direct and cross elasticities) but also in the structural forms of the functions (e.g., as represented by the different cases of SPM-1 and SPM-2).
3. A system should have capability for including explicitly any desired set of level-of-service variables.
4. A system should have a valid procedure for computing equilibrium of supply and demand within the network, considering the interaction of all market segments.
5. To implement recommendation 4 will probably require a direct approach using explicit demand models. It should be implemented in a single integrated system of computer programs. (Although DODOTRANS is one candidate for this, more efficient and satisfactory procedures can certainly be developed. Ultimately, a technique may be developed for computing a valid equilibrium with the indirect approach. Until that time, the direct approach should be used.)
6. The GSM theorems (especially theorem 4) suggest that a single model system could be developed to compute equilibrium in the direct approach by using the explicit form of the GSM:

a. Any of a large variety of specific functional forms for functions such as α and β can be provided; the user can select those he wishes to use (and the corresponding sets

of parameter values) when he makes his run to analyze a particular transportation plan. The same equilibrium computational procedures would be used for every form. (Ideally, the computational procedures should allow simultaneous use of several different forms corresponding to different market segments.)

b. As operational experience with specific functional forms is gained, additional special-purpose algorithms can be developed and added to the system for more efficient computation of equilibrium for specific forms of demand models.

c. The availability of alternative forms in a single model system would allow the analyst to do sensitivity studies of alternative models as well as of alternative parameter values.

7. In the same model system, options should be provided to compute equilibrium with an indirect approach by using the sequential implicit form of the GSM:

a. As a minimum, the modified UTMS forms (SPM-1, SPM-2, and SPM-3) should be provided in the indirect approach (obviously, levels of service would thus enter into every step of the indirect approach).

b. As an alternative, the direct approach can be used, and the parameters δ of a general form such as SPM-1 can be used by the analyst to control intermediate totals (e.g., V_{k1} and V_k).

8. The same model system should also have the pivot-point capabilities suggested by the elasticity properties of the GSM (1, 28, 29).

9. A major program of research should be mounted to develop specific demand models for a variety of different market segments and under different conditions of urban area life style and transportation system. Theoretical research should explore the properties of various families of specific models. Empirical research should attempt to get results that can be generalized across many urban areas through careful research design. The GSM may be considered as one hypothesis (of many) to be tested and can serve as a framework for development of specific models.

10. Research should be undertaken to develop efficient procedures for computing equilibrium in networks. Priority should be given to direct approaches.

Immediate Actions

The recommendations presented in the preceding section will likely take a few years to accomplish, even if a decision by those in control of the resources were taken today. The theoretical results also suggest specific practical steps that can be taken almost immediately:

1. Exploration should be undertaken of immediate modifications that can be made to existing model systems (the FHWA-UTMS packages, DODOTRANS, and others) to implement capabilities of analyzing a single multimodal network, handling a range of level-of-service variables, iterating all steps in the indirect approach to achieve convergence to equilibrium, having level of service influence each step—including generation and distribution as well as modal split and assignment—in a consistent manner, and representing the behavior of different market segments.

2. Modifications of existing systems should be developed to implement SPM-3, SPM-2, and SPM-1 to obtain acceptable forms of UTMS-like capabilities.

3. Immediate efforts should be initiated to develop pilot or experimental model systems that compute equilibrium in a direct approach with the GSM.

CLOSING

This presentation has ranged from some fairly abstract to some relatively practical issues. We invite discussion and debate on both the theoretical results and the policy recommendations arising from those results.

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USE OF TOPAZ FOR GENERATING ALTERNATE LAND USE SCHEMES

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A technique for the optimum placement of activities in zones was developed initially for use in Melbourne, Australia. The intent of the technique was to generate land use allocation schemes that were optimal according to some preset objectives. In this paper, the technique is employed for Blacksburg, Virginia, both to test a proposed land use scheme and to use this scheme as a basis for finding better arrangement patterns. Capital cost data for water, sewerage, local streets, electricity, and individual building units for each zone in the study area were used in conjunction with travel cost and land value information to derive overall cost minus benefit values for the schemes indicated above. Results seem to suggest that the technique is capable of creating worthwhile alternatives to the proposed schemes, although additional effort needs to be devoted to improving the variety of output information that can be generated by the technique.

•THE DEVELOPMENT of land use plans for an urban area usually is a time-consuming and an expensive process. As a result, the planner often is limited to investigating only a few alternate land use development schemes, and these investigations generally are rather quick and rough. In many instances, the best that can be done is to draw a few sketch plans and determine their probable impacts subjectively. Moreover, the planner is almost always working with the anxiety that more time spent on broad-scale plan development means less time available for the arduous task of completing the final plan in detail.

The planner would be greatly aided if he had a fairly rapid technique that, with a given set of data, would generate and determine some of the consequences of various land use schemes. In those cases where it is possible for him to be more specific about his objectives, he would also be aided by a technique that would generate schemes that were fairly close to optimal in terms of these objectives. Quite obviously, though, the complexity of most urban areas would hinder the development of techniques that would provide anything but first-order approximations of consequences. But then, first-order approximations may be more than adequate for initial sketch planning.

TOPAZ (technique for the optimum placement of activities in zones) seems to be of benefit in the sketch-planning stage. It was first used in the Melbourne, Australia, metropolitan area. The basic idea behind TOPAZ, as envisioned by Brotchie, Sharpe, and Toakley (1, 2, 3), was to use readily available mathematical allocation schemes to organize land use development in an urban area. Initially, the minimization of public service and travel was the main siting objective, although it was recognized from the start that costs certainly were not the only items of concern. Public service costs included those for water, sewerage, local streets, hospitals, and schools, to name a few. A prediction was made of how much would be needed by 1985 for high- and low-density residential land and industrial land. TOPAZ then was employed to determine where to allocate the needed land use areas so as to minimize the public service and

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travel costs. All solutions were constrained so that areas available for development in each zone of the city were not "filled" above capacity. The minimum cost allocations obtained via TOPAZ proved to have some interesting ramifications for development policies in Melbourne.

USE OF TOPAZ IN BLACKSBURG

An endeavor similar to the one in Melbourne was launched in Blacksburg, a small but expanding town of 22,000 people (including students) in southwest Virginia. This endeavor was intended as a prototype but actually may prove to have some worthwhile practical benefits, for Blacksburg is at present involved in a court case related to attempts to annex part of the adjacent county. (Service costs, of course, are important items in annexation considerations, especially for small rural communities.)

The town was divided into 61 zones, which are shown in Figure 1 along with existing land development. (Some of the 61 zones were combined later in the analysis and thus are not shown in Figure 1.) Zonal delineation was done, as in many planning studies, primarily on the basis of slope of the land, depth of bedrock, soil type, availability of existing utilities, existing land use development, natural drainage areas, and man-made boundaries (e.g., US-460 bypass). Figure 2 shows the land slopes, and Figure 3 shows the proposed water system improvements for the area. These are presented to give some idea of the kind of information needed as input to TOPAZ.

The data for the zonal delineation study also were utilized in part in the determination of the per acre establishment capital costs and benefits. Costs were divided into 5 categories: building unit, water, sewerage, local streets, and electricity (supplied by local developers and the town). These costs vary, of course, according to land slope, the need to excavate in bedrock, soil type, nearness to existing services, and so on. For example, in zone 1, which has a slope range of 12 to 20 percent and bedrock very close to the surface, it was determined from conversations with town officials and local land developers that public service capital costs would be about 120 percent higher than those in the lowest cost zones. Costs per acre also varied with the type of activity or land use being considered. In the case of Blacksburg, 16 activities were employed (Table 1). Examples of the costs per acre used in Blacksburg are given in Table 2.

The determination of benefits naturally proved to be rather difficult. Our interest was in indicating the benefits an activity or land use type would receive from being located in places that had certain amenities, such as a good view of the mountains, nearness to other activities, and good landscaping. As a very rough measure of all these, we used land values. We do not feel that this measure is entirely adequate, but at least we have attempted to include some representation of items other than costs. Typical land values are also given in Table 2.

Travel costs are also taken into account in TOPAZ. A gravity model is used to make estimates of zone-to-zone movements based on existing and future amounts of each activity in each zone. From a mathematical standpoint, the inclusion of the gravity model makes the determination of the optimal allocation of activities a very difficult matter. However, the main advantage of TOPAZ is that it incorporates an iterative solution scheme that is very fast and gives solutions that, although not necessarily global optima, seem to be very close. The Appendix contains a mathematical formulation and small numerical example of TOPAZ.

The main elements in Blacksburg's transportation system have been surveyed and coded in a manner similar to that done in most large-scale transportation studies. Interzonal travel costs for each daily trip predicted by means of the gravity model were obtained by summing costs on each link on the minimum time path between zones. These costs then were multiplied by the expected repetitions of that daily trip for each year up to and including the horizon year. Overall travel costs probably could be expected to be relatively low because we assumed a repetition rate of 200 trips per year and a cost per mile of $0.065. (The number of trips is not too low because Blacksburg is a university town, and there are many times during the year when the 13,000-member student body is not in full attendance. Although low, the cost-per-mile figure can be adjusted and tested via sensitivity analysis using TOPAZ.) The horizon year was 1990.

Figure 1. Zones and existing development.

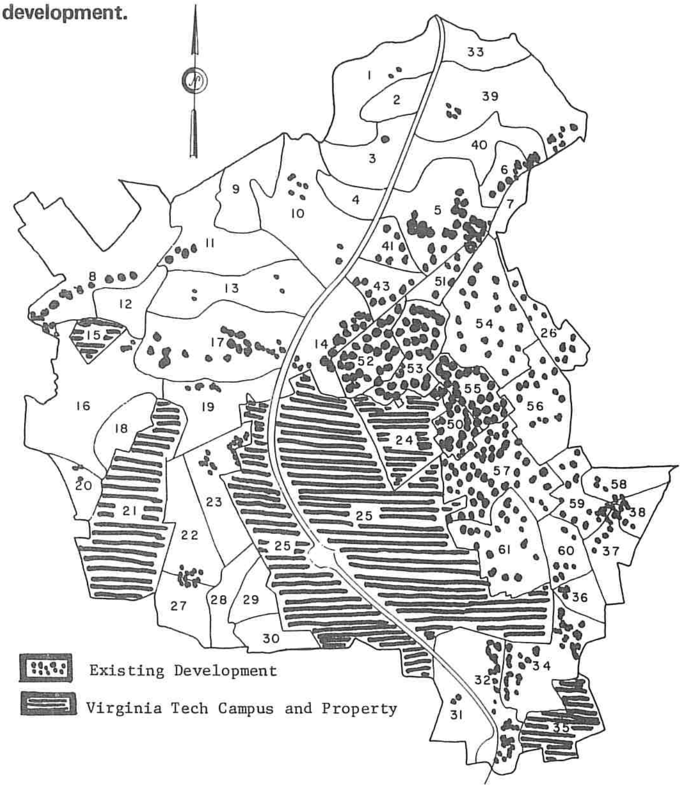


Figure 2. Slope of land.

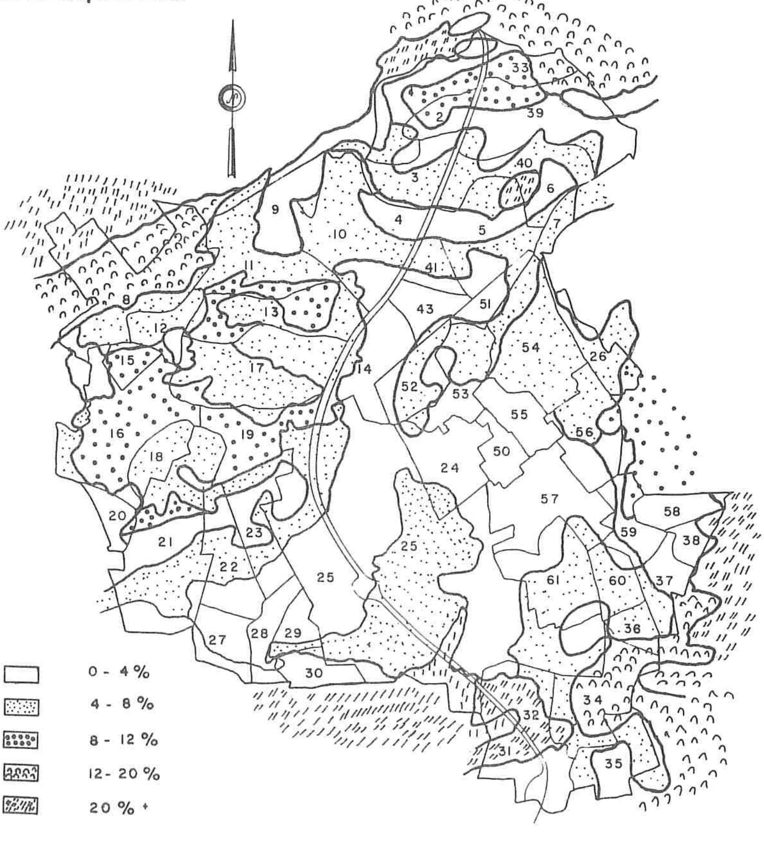


Figure 3. Priorities of water system improvements.

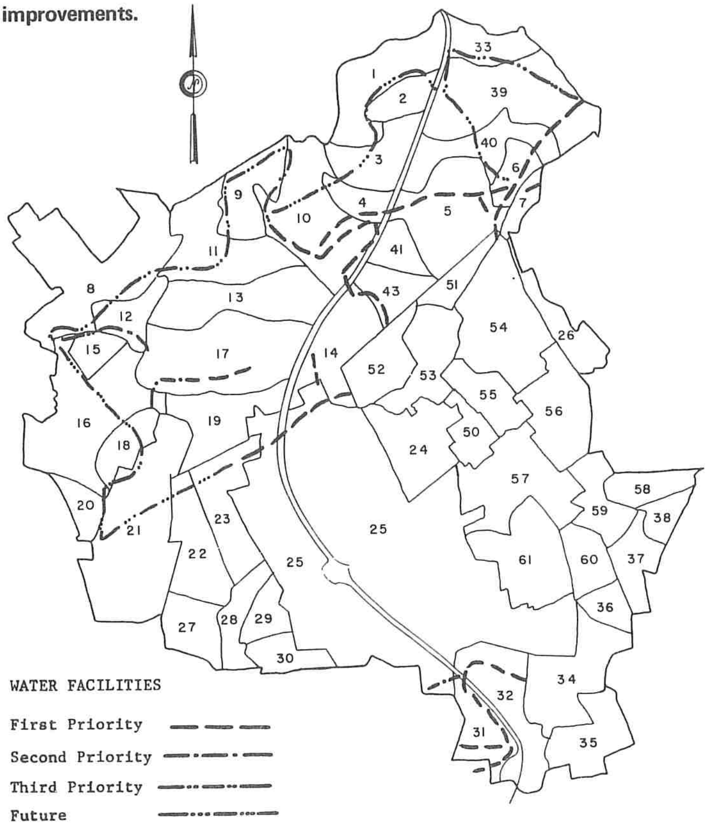


Table 1. Land use activity codes.

Activity	Code	Activity	Code
Single-family houses	1	Town parks	9
Apartments	2	Primary schools	10
Town houses	3	Secondary schools	11
Planned unit development	4	Public and semipublic land	12
Mobile homes	5	Industry	13
Convenience commercial	6	Streets[a]	14
Regional commercial	7	University	15
Neighborhood parks	8	Undeveloped land	16

[a]This category was not actually used in the analyses because all areas were gross areas, that is, including local streets and alleys.

Table 2. Establishment cost and benefit values.

Cost or Benefit (dollar/acre)	Zone	Single Family	Apartment	Town House
Building unit cost	1	64,000	184,000	144,000
	2	54,000	154,000	120,000
	3	55,000	155,000	121,000
	4	54,000	154,000	120,000
Sewerage system capital cost	1	2,960	3,460	3,460
	2	1,810	2,310	2,310
	3	1,810	2,310	2,310
	4	1,810	2,310	2,310
Amenity benefit[a]	1	-1,500	-2,250	-2,250
	2	-1,500	-2,250	-2,250
	3	-1,500	-2,250	-2,250
	4	-2,000	-3,000	-3,000

[a]Land values. The minus signs indicate negative costs, that is, benefits.

The remaining sets of information required for input to TOPAZ are the estimates of areas available for development in each zone and the estimates of the areas of each activity (land use) required by the horizon year. The first set of estimates is obtained fairly readily through a typical land use survey. However, there is a definitional question as to what constitutes land available for development. Should land with a slope greater than 20 percent be considered available? Is land already dedicated for an industrial park or zoned commercial really available for other uses? These and similar questions become quite perplexing. Our approach has been to assume that almost all vacant land is available. By acting in this manner, we leave ourselves in a flexible position, for we can come back later if we so desire, incorporate restrictions of various sorts (e.g., zoning and general policy), and determine the increased costs brought about by these restrictions. In this way, we are able to set up trade-off situations where we can ask, for example, whether the increased costs occasioned by, say, a certain zoning ordinance are more than offset by the anticipated benefits (excluding land values).

The second set of estimates, the amounts of land use areas needed by the horizon year, is perhaps the least reliable input to TOPAZ. These areas are obtained by taking the forecast population figure for the overall region and applying certain proportions to it. The population of Blacksburg plus the student body is expected to grow from 22,000 people at present to 40,000 people within the next 20 years. Of the increase of 18,000 people, 9,000 are expected to be students and 9,000 permanent residents. In this latter group, it is anticipated that 6,000 will wish to live in single-family houses. Based on 3.2 persons per family and 3 single-family units per gross acre (including streets and other services), about 626 acres of single-family homes will be needed. Similar reasoning is employed to obtain estimates of the other activity areas required. The amounts of these areas could vary somewhat, of course, especially since we are assuming currently accepted development standards, current zoning density restrictions, and a similar pattern of demands for land use as at present. But again, we can do some sensitivity analyses to see how land allocations may change. We could, for instance, determine what happens when the demand for town houses increases while that for single-family houses decreases.

RESULTS OF THE APPLICATION TO BLACKSBURG

A series of runs were made with TOPAZ and the Blacksburg data. It will not be possible to report on all the results here, but we will attempt to highlight the important ones.

TOPAZ requires that a feasible solution be assumed initially. This solution then is upgraded to an optimal one (or close thereto). We started with a solution that the town's planner particularly desired to test because it designated growth in many of the areas for which the town anticipated providing water and sewer extensions. The initial solution is shown in Figure 4. It includes, predominantly, incursions to the northwest side of town in zones 9, 10, 11, 13, and 17. TOPAZ automatically "costs out" all initial solutions; the following costs and benefits were obtained:

Benefit or Cost	Millions of Dollars
Establishment benefits	-3.9
Building unit costs	66.2
Water system costs	2.2
Sewer system costs	2.1
Local street costs	3.1
Electric system costs	0.8
Travel costs	19.5
Total	90.0

The size of the benefit and cost items should be of interest at this point. The 3 items of the largest magnitudes (establishment benefits and building unit and travel costs) are the ones for which the town probably would have the least concern because

it does not have to pay for these directly. Travel costs are about 22 percent of the total, a relatively low figure because most travel is for short distances in a small town. (Travel costs could be more substantial in a large city, however; this finding was borne out in Melbourne to some extent.) The costs of direct concern to the town total about $8.2 million.

The optimal land use pattern generated by TOPAZ starting with the initial solution (Fig. 4) is shown in Figure 5. The benefit and cost figures for this pattern are as follows:

Benefit or Cost	Millions of Dollars
Establishment benefits	-5.8
Building unit costs	65.9
Water system costs	2.0
Sewer system costs	1.8
Local street costs	2.9
Electric system costs	0.7
Travel costs	16.6
Total	84.1

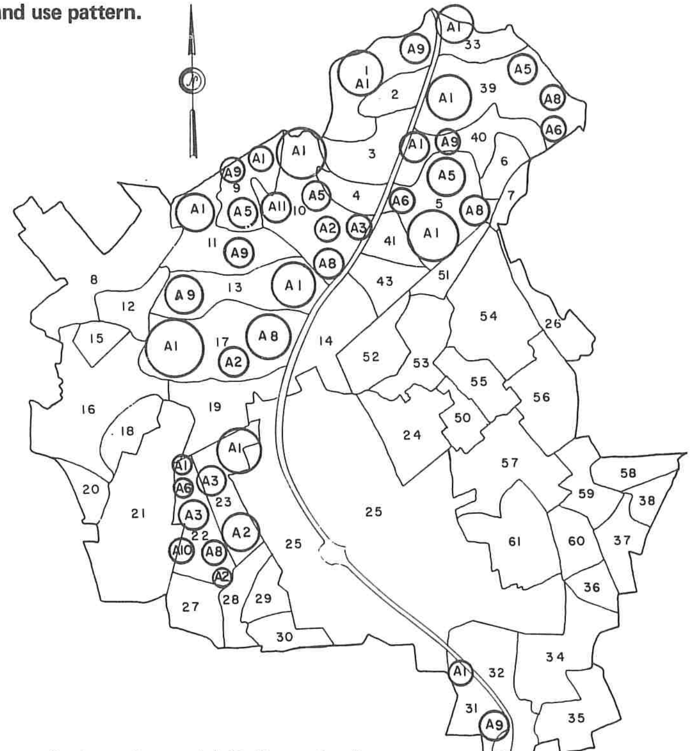
Total overall costs have been reduced $5.9 million for the initial solution, but the makeup of component changes is of interest. Establishment benefits have risen $1.9 million, indicating that land uses have been placed in areas with more amenities. Travel costs have decreased by $2.9 million, while costs of direct concern to the town have decreased only $0.6 million. Thus, it appears that the town's anticipated strategy of locating some major water and sewer mains on the northwest side will increase their direct costs only slightly but will put an added travel burden on the public and perhaps induce people to go where their "benefits" would not be quite so great. These results are borne out by a close survey of the zonal allocations shown in Figures 4 and 5. The proposed expansion shown in Figure 4 to the northwest—zones 9, 10, 11, 13, and 17—is not shown in Figure 5. Instead, much more use is made of the closer in, currently built-up zones to the north and east of town. This TOPAZ-generated alternative obviously presents a quite different land use development scheme from the one currently being considered. (Interestingly enough, this TOPAZ scheme does not allocate much land to areas being considered for annexation by the town.) A warning is in order, however. There may be other benefits not taken into account in TOPAZ that more than make up for the additional costs (and lack of benefits) to be incurred in the initial solution. Yet, the trade-offs are more explicit now: Are the additional benefits not considered in TOPAZ worth the extra $5.9 million in costs and foregone land value benefits, $0.6 million of which is in direct costs to the town? Perhaps only the political process can answer this question. But at least the consequences are clearer, and new and apparently worthwhile alternatives have been generated.

ADDITIONAL RESULTS

To provide added perspective to the results given above and to demonstrate some of the versatility of TOPAZ, we have made an extra set of analyses based on the following objectives: maximize overall costs-benefits, minimize direct town costs, maximize direct town costs, minimize travel costs, and maximize travel costs.

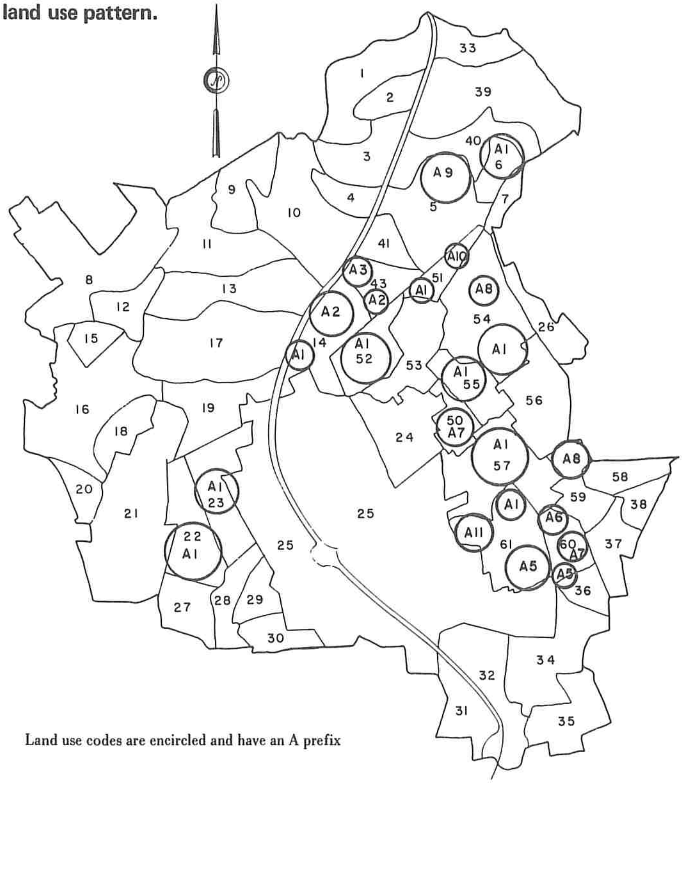
The purpose of the first analysis was to see what the worst land use pattern would be and thereby to provide both a datum by which to judge schemes with intermediate cost consequences and an indication of where growth definitely ought not to go. The resultant maximum value was $107 million, of which $86.6 million was for establishment costs minus benefits and $20.4 for travel. We now can see that the town's anticipated scheme would be about 25 percent of the way toward the worst case on an overall cost-benefit scale. The worst land use pattern itself (not shown) is somewhat as one might expect; activities are allocated to the most expensive peripheral zones.

Figure 4. Initial land use pattern.



Land use codes are encircled and have an A prefix

Figure 5. Optimal land use pattern.



Land use codes are encircled and have an A prefix

The scheme with the lowest direct costs to the town would represent an expenditure of $7.5 million on the 4 direct-cost items. The corresponding maximum scheme would entail an expenditure of $11.3 million. The town scheme, as could be expected, is fairly close to the low end of this range. The minimum and maximum travel cost schemes give cost figures of $15.4 million and $20.7 million respectively. These figures tend to affirm the earlier argument that the town scheme ($19.5 million in travel) encourages longer trip-making and, thus, is more expensive along that line, especially because the areas to be developed under that plan do not have particularly good access. Thus, the town scheme falls toward the top of the range in this respect. On the other hand, the TOPAZ-generated optimal scheme falls near the minimum. These results all are of some consequence but should be judged only in connection with the assumptions implied in TOPAZ.

CRITIQUE OF TOPAZ

There are many assumptions employed in TOPAZ that have not been tested to any great degree (if at all). There are also many areas where the technique can be improved simply with additional time and effort. We will briefly list and discuss items in each of these 2 classes.

We must first recognize the fact that the land use controls needed to implement various results from TOPAZ are, at least in most American cities, almost nonexistent. Nonetheless, urban plans in general have almost always been advisory in nature so that schemes generated by TOPAZ are not likely to be more disadvantageous in this respect. However, we anticipate that, as new towns become more prevalent and as more thought is given to national and statewide land use policies, new and stronger land use controls may be forthcoming that could aid in implementing results from TOPAZ.

Another major drawback with TOPAZ is that it is focused almost entirely on physical planning. Yet, we know that often there is a strong connection between the physical, the economic, the social, and the political. Only by considering land values have we even started to make a rough approximation of private sphere economic, social, and political gains and losses. Still, there are many problems involved with our present approach. First, we have made the mistake of double counting benefits and costs because part of the benefits inherent in land values are those attributable to accessibility. This factor is already considered somewhat through reduced travel costs, but the extent of double counting generated here is unknown. A second aspect of the benefits component is that locational benefits, at least in the economic sense, refer to what a person is willing to pay (demand) for land, not necessarily what he actually pays. We equated actual payments to benefits because we had no clear picture of the demand curve for land in Blacksburg.

Another problem involves capital improvements for water purification plants, sewage-treatment plants, and secondary and primary roads. It would seem desirable to have TOPAZ determine where these major facilities should go to minimize even further the difference between costs and benefits, but such a process is not as yet possible. It is thus necessary to assume some levels of improvements in these facilities (and their locations) and determine the costs that would result at these levels. We have assumed no major facility changes in this regard, mainly because we would have gone to considerable effort to estimate new costs per acre in many zones if facilities of these types were added.

A final major assumption in TOPAZ has some interesting political ramifications. What happens if, say, a water line is constructed through vacant land to an outlying property? Should the cost per acre for water in that outlying zone include the full cost of the line when, eventually, some people will settle in between and possibly reimburse those on the periphery? We have assumed that the reimbursement would occur so that costs per acre would not depend on expenditures for facilities outside the zone itself. (The town currently is evaluating its present policy that requires those on the periphery to pay the full cost and makes no stipulation for eventual reimbursement.)

There are also short-range deficiencies in TOPAZ that can be readily adjusted with more effort. We would make the following changes in this endeavor:

1. Include public service operating costs;
2. Consider economies of scale for public services, that is, lower the service costs per acre when a greater number of acres are developed;
3. Specify costs on an annual basis (involving potential problems in determination of interest rates and service lives);
4. Include, where reasonable, upper and lower limits on the size of any activity allocated to a zone (theoretically it is possible for TOPAZ to allocate, say, 1 acre of regional commercial land use to a zone, and, although this certainly is unrealistic, the chance of its happening is slight);
5. Consider certain parts of the urban area as being available for redevelopment;
6. Include trips with one or both ends external to the study area;
7. Increase sophistication of the gravity and trip generation models, perhaps with a 3-purpose breakdown;
8. Include modal choice and trip assignment; and
9. Include a process for staging improvements.

These changes, along with additional sensitivity analyses of assumed parameter values (e.g., future population levels or number of building units per acre) should help refine TOPAZ into a more workable tool for sketch-plan generation and analysis.

FINAL REMARKS

The application of TOPAZ was intended to be mostly a prototype endeavor. It soon became clear, however, that the results generated by TOPAZ could be of significance to Blacksburg's development policy, especially in regard to annexation and future extensions of water and sewer lines. It is to these kinds of general policy questions that TOPAZ is directed, and, as such, we feel that it is providing some worthwhile alternatives for political consideration. Nonetheless, TOPAZ needs much in the way of refinement.

ACKNOWLEDGMENTS

We would like to thank Peter Snyder of Snyder/Hunt Builders, William Price of Price Realty, Phillip Pappas of AIA, and Gordon Dixon, a planner for the town of Blacksburg, for their aid in this project, especially insofar as cost data acquistion was concerned. Samuel R. Seward, Dennis Couture, and Fazil Najafi provided valuable assistance, particularly in analyzing and coding the transportation network. All conclusions made are strictly those of the authors and not necessarily those of the town of Blacksburg. The initial solution used in TOPAZ, designated as "the town scheme," is only one of several being considered by the town and has no official status.

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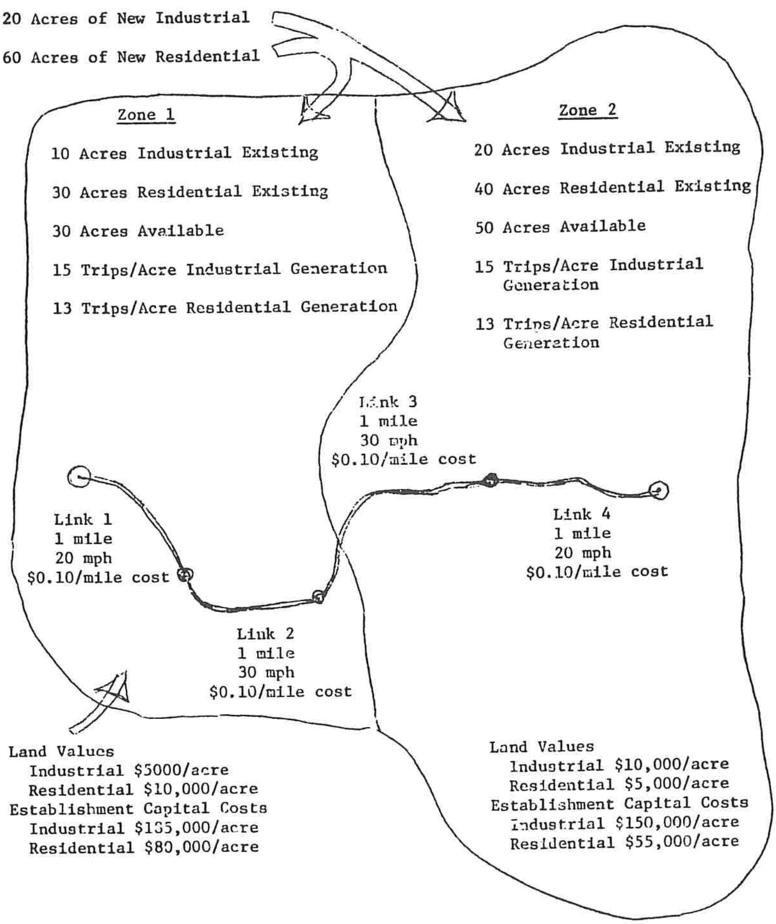
APPENDIX

MATHEMATICAL DESCRIPTION AND EXAMPLE OF TOPAZ

This example is a 2-zone case shown in Figure 6. The centroids of the 2 zones are connected by a highway having 4 distinct links. The lengths of these links are all 1 mile, but the speeds are higher on the middle 2 links (30 mph) than on those connected directly to the centroids (20 mph). Travel costs are the same on all links (10 cents/ mile).

Zone 1 is smaller than zone 2, having only 70 acres of land with 30 available for future development. Zone 2 has a total of 110 acres with 50 available for development. It is desired to put 20 acres of industrial and 60 acres of residential land somewhere within the 2 zones. Land values are somewhat equally distributed among activities and zones, whereas service capital costs are $35,000/acre higher for industrial sites and

Figure 6. Representation of 2-zone example.



\$25,000/acre higher for residential sites in zone 1 than in zone 2. It is desired to find the allocations of new activities to zones so as to minimize total travel plus establishment costs minus benefits. All new activities must be allocated, and the amount of area available for development in each zone cannot be exceeded.

The notation used is as follows:

X_{ij} = amount of activity i allocated to zone j, acres;
E_{ij} = existing amount of activity i in zone j, acres;
A_i = future amount of activity i to be allocated, acres;
B_j = area available for development in zone j, acres;
c_{sij} = unit establishment benefits or capital costs for service s for activity i in zone j, dollars/acre;
C_{ij} = total establishment costs-benefits for locating activity i in zone j, dollars/acre;
PR_i = daily vehicular trip production rate for activity i, vehicles/day/acre;
AT_i = daily vehicular trip attraction rate for activity i, vehicles/day/acre;
S_ℓ = speed over link ℓ, mph;
L_ℓ = length of link ℓ, miles;
P_{jk} = set of links on the minimum time path from zone j to k;
T_{jk} = minimum highway travel time from zone j to k, min;
M_{jk} = distance over minimum highway travel time path from zone j to k, miles;
d = number of repetitions of daily trips in a year;
y = length of planning horizon, years;
pm_ℓ = vehicular cost to travel over link ℓ, dollars/mile;
z = sum total of all travel costs and establishment costs-benefits, dollars;
z' = value of the objective function of the linear "transportation problem," dollars; and
K_{jk} = cost over the planning period for a repetitive trip from zone j to k, dollars/daily trip.

The input information for the example is listed below.

1. Activity descriptions: Activity 1 is industrial, and activity 2 is residential.
2. Existing activities, acres: E_{11} = 10, E_{12} = 20, E_{21} = 30, and E_{22} = 40.
3. Areas available for development, acres: B_1 = 30 and B_2 = 50.
4. Areas of activities needed to be developed, acres: A_1 = 20 and A_2 = 60.
5. Trip production and attraction rates, vehicles/acre/day: PR_1 = 15, PR_2 = 13, AT_1 = 15, and AT_2 = 13.
6. Benefits (land values), dollars/acre: c_{111} = -5,000, c_{112} = -10,000, c_{121} = -10,000, and c_{122} = -5,000.
7. Public service capital costs, dollars/acre: c_{211} = 185,000, c_{212} = 150,000, c_{221} = 80,000, and c_{222} = 55,000.
8. Link speeds, mph: S_1 = 20, S_2 = 30, S_3 = 30, and S_4 = 20.
9. Link lengths, miles: 1.
10. Links on minimum time paths: $P_{11} = \{1\}$, $P_{12} = \{1, 2, 3, 4\}$, $P_{21} = \{1, 2, 3, 4\}$, and $P_{22} = \{4\}$.
11. Vehicular travel costs, dollars/mile: 0.10.
12. Other information: d = 200 trip repetitions/year and y = 20 years.

Some preliminary calculations are needed before the actual TOPAZ equations are given. First, we must sum the component establishment costs and benefits to get a total for each activity and zone. Thus,

$$c_{ij} = \Sigma\, c_{sij}, \text{ all } i, j \tag{1}$$

The travel distances and times between zones are found by adding the link distances and times respectively over the minimum time path between the zones (specified beforehand):

$$M_{jk} = \sum_{\ell \in P_{jk}} L_\ell, \text{ all } j, k \tag{2}$$

and

$$T_{jk} = 60 \sum_{\ell \in P_{jk}} L_\ell / S_\ell, \text{ all } j, k \tag{3}$$

Travel costs over each minimum time path are computed by taking into account the number of times each daily trip (found from the gravity model incorporated in TOPAZ) is repeated within each year in the time span up to the planning horizon date. Therefore,

$$K_{jh} = yd \sum_{\ell \in P_{jk}} pm_\ell L_\ell, \text{ all } j, k \tag{4}$$

Using the 4 equations given above, we can calculate the values for C_{ij}, M_{jk}, T_{jk}, and K_{jk}. The results are listed below.

1. Total establishment costs-benefits, dollars/acre: $C_{11} = 180{,}000$, $C_{12} = 140{,}000$, $C_{21} = 70{,}000$, and $C_{22} = 50{,}000$.
2. Distances between zones, miles: $M_{11} = 1$, $M_{12} = 4$, $M_{21} = 4$, and $M_{22} = 1$.
3. Travel times between zones, min: $T_{11} = 3$, $T_{12} = 10$, $T_{21} = 10$, and $T_{22} = 3$.
4. Travel costs between zones, dollars/daily trip: $K_{11} = 400$, $K_{12} = 1{,}600$, $K_{21} = 1{,}600$, and $K_{22} = 400$.

TOPAZ has as its objective the minimization of the combination of overall travel costs and the establishment costs minus benefits. Travel between zones (and travel costs) is determined with the aid of a gravity model. Establishment costs and benefits are input values. The formulation for TOPAZ can be presented as follows:

$$\min z = \sum_i \sum_j C_{ij} X_{ij} + \sum_j \sum_k K_{jk} \sum_i PR_i (X_{ij} + E_{ij}) \frac{\sum AT_i (X_{ik} + E_{ik}) \frac{1}{T_{jk}^2}}{\sum_n \sum_i AT_i (X_{in} + E_{in}) \frac{1}{T_{jn}^2}} \tag{5}$$

$$\sum_j X_{ij} = A_i, \text{ all } i$$

$$\sum_i X_{ij} = B_j, \text{ all } j$$

$$\sum_i A_i = \sum_j B_j$$

$$X_{ij} \geq 0, \text{ all } i, j$$

The first term in the objective function is the total establishment costs minus benefits. The second term is the gravity model equation; the daily trips between zones j and k multiplied by the travel cost K_{jk}. The term $\Sigma PR_i (X_{ij} + E_{ij})$ takes the existing activity of type i in zone j, E_{ij}, and adds it to the allocated amount, X_{ij}.

This total for activity i then is multiplied by the daily vehicular trip production rate for that activity PR_i, which gives the number of trips produced by that activity in the zone. Summing the trips produced by all activities in the zone then gives the total trips produced by the zone (or, stated in terms of the gravity model, the trip productions of zone j). Similar reasoning applies in the formation of the trip attractions terms, $\sum_i AT_i (X_{ij} + E_{ij})$. After productions and attractions have been determined, the gravity model is used to predict the trips between each pair of zones. This is done by dividing the trip production for zone j according to the trip attractions and squared travel times of a zone k relative to all other zones (all n).

The first set of constraints in Eq. 5 ensures that the future amounts of each activity are allocated. The second set ensures that the exact acreage of land available in each

zone is used up. It should be noted here that vacant land, total developable land still remaining after future acreages of all activities are taken into account, is also considered to be an "activity" to be allocated. (This will not be shown in the upcoming example because of the great increase in computation required.) By viewing the left-over land in this manner, we thus can fulfill the third type of constraint in Eq. 5 (which is needed because of the particular computer code employed).

Equation 5 cannot be easily solved for the X_{ij}'s because they occur both in the numerator and denominator of the objective function (and in a nonlinear fashion in the numerator). As a consequence, TOPAZ involves an iterative solution procedure in which enough feasible values of the X_{ij}'s are assumed initially to make the objective function in Eq. 5 linear throughout. This linear version is the standard "transportation problem," which can be solved rapidly with available algorithms and computer codes. [We have utilized a code based on the algorithm by Ford and Fulkerson (9).] The X_{ij}'s that are the solution to the transportation problem are substituted for the initially assumed values in Eq. 5, and this process creates another transportation problem. This procedure continues until the lowest value for z is noted.

As an example of this iterative process involved in TOPAZ, let us substitute values from the input information given earlier into Eq. 5. As indicated in the previous paragraph, it is also necessary to assume or guess a feasible solution to Eq. 5. This is relatively easy, even for large problems, and often the solution the planner thinks is best can be used here. For instance, we can assume that $X_{11} = 20$, $X_{12} = 0$, $X_{21} = 10$, and $X_{22} = 50$ acres. These values meet the constraints since $X_{11} + X_{12} = A_1 = 20 + 0 = 20$, and so on. At this point, if desired, we can actually "price out" this assumed solution. Thus,

$$z = 180{,}000(20) + 140{,}000(0) + 70{,}000(10) + 50{,}000(50)$$

$$+ \frac{400[15(20+10) + 13(10+30)][15(20+10) + 13(10+30)]1/(3)^2}{[15(20+10) + 13(10+30)]1/(3)^2 + [15(0+20) + 13(50+40)]1/(10)^2}$$

+ costs for travel from zones 1 to 2, 2 to 1, and 2 to 2

Out of these calculations we find that, for the proposed solution,

Establishment costs-benefits	\$6,800,000
Travel costs	1,146,540
Total	\$7,946,540

This total cost figure will provide a good datum to judge the gains registered through TOPAZ.

Now to continue with TOPAZ itself, we substitute the X_{ij}'s for the proposed solution given above into the terms for the trip attractions (but not for the productions) in Eq. 5. Thus, the objective function for that equation becomes

$$z' = 180{,}000\, X_{11} + 140{,}000\, X_{12} + 70{,}000\, X_{21} + 50{,}000\, X_{22}$$

$$+ \frac{400[15(X_{11}+10) + 13(X_{21}+20)][15(20+10) + 13(10+20)]\,1/(3)^2}{[15(20+10) + 13(10+20)]1/(3)^2 + [15(0+20) + 13(50+40)]1/(10)^2} \qquad (6)$$

+ the other 3 travel cost terms

This objective function now is linear; for example, the expanded gravity model term in Eq. 6 can be reduced to $5{,}185\, X_{11} + 4{,}493\, X_{21}$. If all the calculations needed for the example were carried out, we thus would find that Eq. 5 becomes

$$\min z' = 188{,}451\, X_{11} + 146{,}869\, X_{12} + 77{,}324\, X_{21} + 55{,}952\, X_{22} \qquad (7)$$

$$X_{11} + X_{21} = 30$$
$$X_{12} + X_{22} = 50$$
$$X_{11} + X_{12} = 20$$
$$X_{21} + X_{22} = 60$$
$$X_{ij} \geq 0, \text{ all i, j}$$

Equation 7 is in the form of the standard transportation problem and can be solved rather easily. The solution in this case would have $X_{11} = 0$, $X_{12} = 20$, $X_{21} = 30$, and $X_{22} = 30$ acres.

To see whether this solution brings any improvement in the objective function of Eq. 5, we could substitute the X_{ij}'s in all places in the objective function. Thus,

$$z = 180{,}000(0) + 140{,}000(20) + 70{,}000(30) + 50{,}000(30)$$
$$+ \frac{400[15(0 + 10) + 13(30 + 20)][15(0 + 10) + 13(30 + 20)]1/(3)^2}{[15(0 + 10) + 13(30 + 20)]1/(3)^2 + [15(20 + 20) + 13(30 + 40)]1/(10)^2}$$
$$+ \text{ terms for the other 3 travel costs}$$

We then would find that

Establishment costs-benefits	\$1,144,950
Travel costs	6,400,000
Total	\$7,544,950

Here we can see a decrease in costs of about \$400,000 from the solution assumed at the beginning.

Future iterations with TOPAZ may prove to be even more useful in reducing z. To test this, and to show how the next iteration is set up, we will present one more round. To start, we substitute the solution variables from the previous iteration into the trip attraction terms of the objective function of Eq. 5. This becomes

$$z' = 180{,}000\ X_{11} + 140{,}000\ X_{12} + 70{,}000\ X_{21} + 50{,}000\ X_{22}$$
$$+ \frac{400[15(X_{11} + 10) + 13(X_{21} + 20)][15(0 + 10) + 13(30 + 20)]1/(3)^2}{[15(0 + 10) + 13(30 + 20)]1/(3)^2 + [15(20 + 20) + 13(30 + 40)]1/(10)^2} \quad (8)$$
$$+ \text{ the other 3 travel cost terms}$$

This equation in toto reduces to the linear relation

$$z' = 188{,}612\ X_{11} + 146{,}809\ X_{12} + 77{,}463\ X_{21} + 55{,}900\ X_{22} \quad (9)$$

Equation 9 is not significantly different from the one in the previous iteration so that, as it turns out, the solution variables are identical. TOPAZ has reached its stopping point (although in large-scale applications it usually proceeds 4 or 5 iterations before stability at a lower limit is noted). The ultimate solution variables thus have been obtained; $X_{11} = 0$, $X_{12} = 20$, $X_{21} = 30$, and $X_{22} = 30$ acres. The z value of \$1,544,950 may not be a global optimum but, from all indications, it is fairly close.

Insofar as computation times are concerned, we have found TOPAZ to be an extremely fast technique. For the Blacksburg case, there were 61 zones and 16 activities (land uses)—or 976 variables. Computing times for this case on the IBM 360/65 computer in no instance exceeded 3 min and were mostly about 1 min. These times are quoted for 4 iterations with the transportation problem subroutine, the usual number required with TOPAZ in the Blacksburg case.

USE OF RANDOM-SEARCH TECHNIQUE TO OBTAIN OPTIMAL LAND USE PLAN DESIGN

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This paper deals with the problem of land use plan design. An efficient land use plan design model can provide a ready tool to achieve the optimal future plan for an urban area or a region through the satisfaction of the design constraints and at an optimum of public and private costs. After the basic features of a land use plan design model are discussed, the difficulties associated with the existing model procedure are examined. The use of a simple procedure based on a random-search technique is then evaluated. The validity of the random technique is established through a series of small-scale, controlled experiments with hypothetical areas. The controlled-experiment procedure is also used to estimate the plan-effectiveness parameters involving the random method.

•IN RECENT years considerable interest has been directed toward the formulation of mathematical models to describe the process of land use planning and design. Under the general heading of the land use model, there are 2 distinct types of models: design model and simulation model. A land use simulation model is an attempt to describe the process of supply and demand of land for various activity uses in an area over a space of time under a set of public and private decisions. Such a model tends to trace the future land use pattern that will evolve from the existing pattern under certain given conditions. On the other hand, the concept of a land use design model is to create an ideal land use design for an area at some future year—an ideal plan that will minimize the total cost as well as satisfy the community development objectives and design standards. In other words, a land use simulation model attempts to predict what the land use pattern will be, and the design model attempts to depict what the land use pattern should be. To be precise, the fundamental distinction between a land use simulation model and a land use design model is essentially the functional distinction between the positivistic and normative models.

BACKGROUND INFORMATION

Basic Features of a Land Use Design Model

Most of the land use models developed in the past few years are simulation models and are positivistic in nature (1, 4, 7). These models are concerned with the problem of land use forecasting and are used to design only in a trial-and-error fashion. Little attempt has been made, however, to develop a normative model that will provide a target plan for an area. The present study involves the development of a land use design model that will offer a ready tool to achieve the ideal future plan for an urban area or a region at optimum of combined public and private costs. Such an optimal design is effected through the satisfaction of the constraints imposed by a series of predetermined design standards for the elements of the proposed plan. In other words, that plan is chosen from a series of alternative plans that best satisfies both cost and design constraints. The usefulness of such a model is evident from the fact that it will enable practicing planners and engineers to arrive at a desired pattern of future

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land use through a more systematic and expeditious process than that offered by the conventional approach.

A land use design model can be viewed as a design tool for the land use pattern of an area. The total available land is quantified by dividing the plan area into a number of cells and by specifying the size and location of each cell in the area. The design demand is established by the land area required by each of the discrete land use activities or elements, such as residential neighborhoods, schools, hospitals, and parks. These discrete land use elements are termed modules, and the entire land use system of an area is expressed in terms of a set of modules. The basic operation of a land use design model consists of the placement of given modules in the specified cells of the plan area. In the following paragraphs, the functions and definitions of some of the key elements of the model are discussed.

Modules

In the land use plan design process, the modules are the basic building blocks. A module, as it is used in the model, is a discrete design unit expressed as a physical entity, for example, a single-family residential area, a neighborhood commercial area, an industrial area, or a recreational area. Each module unit is expressed in terms of space required for the primary land use activity as well as the secondary service areas necessary for the proper functioning of the activity concerned. For example, a module for a neighborhood commercial center might have as the primary area a building site for the stores and as supporting areas a parking lot, truck-loading facilities, warehouses, internal vehicular circulation space, pedestrian malls, open space and landscaped places, ingress-egress zones, and arterial and collector street rights-of-way. In this way, the entire land use activity of an area can be decomposed into a set of discrete, self-sufficient modular units of uniform functional characteristics. The number of modules that may be located in the area under consideration will depend on the space requirements for various land use activities at the design year. These space requirements are obtained by translating the forecast information of socioeconomic variables such as population and employment on the basis of the design standards that are established by the planning agency of the area under consideration.

Cells

After the module types are defined and the numbers and sizes are established, the plan area is divided into a number of cells—spatial units that have more or less uniform characteristics, such as soil and topography or natural or man-made boundaries. Although the shape of these cells can have almost any pattern, there is a limitation on the size; the smallest cell should be large enough in area to accommodate at least one of the largest modules. The delineation of the cells, therefore, depends on the specification of the modules and their sizes. Because the size of the modules depends on the type of plan design, the size of the cells is also influenced by the level of planning; therefore, the cells for a regional plan would obviously be much larger than the cells for a community plan.

Plan Constraints

After the modules are defined and the cells are delineated, it is necessary to identify the constraints that are associated with the land use plan of the area. These constraints form an essential part of the plan design process, for as a whole they control the feasibility of a plan. They are derived from the general planning objectives and, consequently, from the specified design standards. As model input, these constraints are expressed in mathematical form, either by a binary standard or in terms of quantifiable distances. As an example of a binary standard, a particular constraint might be so established that a specific module can or cannot be placed in certain soil types. On the other hand, other constraints might be of a nature that a specific module must be within or at least a certain measurable distance from other module types, and so on.

In general, these constraints can be classified into 2 major groups: cell-module constraints and intermodule constraints. The cell-module constraints are those that exclude location of specific modules in certain cells consisting of incompatible soil. This type of constraint can also be used to preserve some cell areas for the exclusive employment of specific land uses. For example, the cells of a regional corridor that consists of wildlife habitat, wetlands, forests, and woodlands can be excluded from the module-placement process. The intermodule constraints are specified to ensure spatial accessibility and compatibility among the module units, and they are expressed in terms of spatial distances between modules. For example, a constraint can be established specifying that a school must be within a 2-mile radius from the neighboring residential areas, or a sewage treatment plant must be placed at a distance of at least 1 mile from the closest residential areas.

Site and Linkage Costs

The public and private costs associated with the placement of all land use activities can be broadly divided into 2 basic categories: site development cost, which includes the construction and maintenance costs of the module elements, and linkage cost, which consists of construction, maintenance, and operation costs of facilities such as transportation routes, water and sewer lines, and connections for other public utilities between a pair of module units. The site costs are computed on the basis of the soil type and the type of module unit, and they are expressed as dollars per acre of module size. The linkage costs are dependent on the types of module unit to be linked and on the comparative sizes of these units. The linkage requirements for any pair of module types are determined, and construction and maintenance costs per unit distance of linkage as well as the vehicle operation and road-user costs are calculated. The cost values represent present worth values of all cash flows for an interest rate of 6 percent considered during a period of 20 years.

EXISTING PROCEDURE OF MODULE PLACEMENT

The original attempt to develop a design model for a land use plan of an area was initiated at the Southeastern Wisconsin Regional Planning Commission (3, 5, 6). Although the work of the commission firmly established the potential value of a land use design model, the present form of the model is not adequate to develop realistically an ideal land use pattern for an area. The most serious objection to this model is the technique used in the spatial placement of the activity modules.

In order to locate a module in a cell, the design area is successively divided in half, and module elements are assigned to either of the 2 halves of the partition so as to minimize the combined site and linkage costs in the selected partition. The evaluation of minimum costs is made by means of a hill-climbing procedure, and such an evaluation continues until no improved partition can be obtained by shifting a unit element from one half of the partition to the other half. The entire sequence of partitioning is repeated again and again within each of the halves of the preceding scanning process until all the module elements are assigned to cells in the last partition. In this process of module placement, no module once located in one half of a partition can ever be reassigned to the other half in a later scanning.

The technique, which has been utilized in this model, of set decomposition in a series of binary partitions has an inherent shortcoming in its failure to account for the possibility that a particular module element might have been better placed in a different topographic area after the initial partitioning had placed it earlier in a less desirable area. To remedy such errors, called holistic, the provision of higher value partitions in model operations has been suggested (5). However, such a modification might not be too advantageous for the following possible reasons:

1. Although this modification might be expected to cut the holistic errors to some extent, it would not eliminate the errors completely. For example, in a 3-way or 4-way partitioning process, the possibility of a particular module element being trapped in a certain cell would obviously be less than that in a 2-way or 3-way operation respectively; however, it would still be subject to a certain degree of holistic errors.

2. Even if it were at all possible to establish a level of multi-way partitioning, which would reduce the holistic errors to an insignificant degree, the incorporation of such an improvement in model operation would result in an excessively large number of computations, and this in turn, would increase the computer time to an impractically large amount.

RANDOM-SEARCH TECHNIQUE

The most ideal model operation would be an exhaustive search to develop a series of experimental plans by placing each of the modules in each of the cells and sequentially evaluating the respective costs in order to arrive at an optimal design. Such an operation is practically impossible in a complex system with a large number of cells and modules. However, a probabilistic procedure can be adopted to eliminate the large number of trials required in such an exhaustive search. The random-search technique, as discussed by Brooks (2), can be modified and used in the module-placement process. In this probabilistic procedure, a set of experimental plans is developed through the combination of module-cell arrangement designed in a random fashion. The estimate of the best plan is simply that experimental plan where the random assignment of the module-cell combinations produces the lowest total cost and best satisfies the design constraints.

In applying this technique, one can assume that there is an optimal zone of module-cell combinations, which contains a number of best alternative plans. This optimal zone can be defined a priori by establishing the level of plan accuracy that can be assumed as the proportion of optimal zone plans in the entire space of possible experimental plans. Another element that has to be decided a priori is the probability of "success," or the probability that at least one of the experimental plans is contained in the optimal zone. This probability can be expressed as

$$S = 1 - (1 - a)^n$$

where

S = probability of success of the experiment,
a = level of plan accuracy, or the ratio of optimal zone to total number of possible experimental plans,
$1 - a$ = probability that a trial plan will fail to be made inside the optimal zone, and
n = number of trials required to obtain the best plan with the plan accuracy a and the probability of success S.

Solving the above equation for n, we get

$$n = \log(1 - S)/\log(1 - a) \qquad \begin{cases} 0 \le S \le 1 \\ 0 \le a \le 1 \end{cases}$$

By predetermining the values of S and a, we can obtain the value of n from the equation given above. Table 1 gives the respective values of n for corresponding values of a and S. Even if the optimal zone is assumed to be relatively too small or, in other words, if the number of possible alternative best plans is too small compared to the total number of possible experimental plans, the number of trials required to achieve the best plan with a very high probability of success is not more than 919. Furthermore, the number of trials required to achieve a certain level of plan accuracy as well as probability of success does not depend on the number of module-cell combinations. Therefore, such a technique can be conveniently applied to the land use design problem of a comparatively large area or region.

MODEL ALGORITHM

The operation of the random-placement method as applied to a land use plan design model is briefly discussed in the following steps:

1. Input information is fed to include cell data, site and linkage costs, constraint schedule, and plan effectiveness parameters a and S.
2. A module type is chosen through the use of a uniformly distributed random-number generator. If the module chosen is one that has been used before, more random numbers are generated until an acceptable one is found.
3. A cell is then chosen for this module again through the use of a uniformly distributed random-number generator. If the cell chosen is already occupied with a module, another number is generated until one is found that is unoccupied.
4. Before a chosen module is placed in a chosen cell, a check is made to test whether such a placement violates the site and design constraints. The scanning continues until all the modules are assigned to the cells. At this point one experimental plan has been developed.
5. Site costs for the individual modules are computed, and the total site cost of the plan is obtained.
6. Linkage costs for all required links between the different module types are calculated, and the results are totaled. The total cost of the experimental plan is the combined total site cost and total linkage cost.
7. The entire procedure is repeated for as many trials as necessary to obtain the desired plan accuracy and the probability of success that are both specified as input information.
8. During the iterations, the minimum total cost and the module-cell arrangement that gives this cost is stored as running data. At the completion of the trials, the optimal plan and its cost are printed out.

VALIDATION OF THE PROPOSED TECHNIQUE

Controlled experiments were conducted to test the validity of the random-model algorithm. The results obtained from the random algorithm were compared with the results generated by an algorithm based on the exhaustive-search technique. This was accomplished by considering a number of hypothetical study areas consisting of 10 to 15 cells and a total land area ranging from 1,600 to 2,400 acres. Because of the limitation imposed by the number of iterations required for an exhaustive search (101 or 3,628,800 iterations or number of possible plans for 10 cells and 10 modules, excluding the repetitions of a plan), the study area was not made any larger. Each cell contained a combination of soil types, and no 2 cells had the same soil combination. The experiments were conducted in the following 3 steps:

1. Optimal plan based on site and linkage costs;
2. Optimal plan based on all costs and only positive intermodule constraints; and
3. Optimal plan based on all costs and both positive and negative intermodule constraints.

In each step, experiments were made with various given cell-module combinations. The two parameters a and S involving the plan effectiveness in a random-search technique were also varied over a range. Furthermore, trials were made with the same cell-module combinations and plan effectiveness parameters but with different random-number seed values to initialize the random-number generators.

Both site and linkage costs for the given soil and module types were prepared on the basis of data obtained from the Southeastern Wisconsin Regional Planning Commission. The module definitions and the types of intermodule constraints were kept the same as those used in the existing model. This was done in order to coordinate the present research with the commission's ongoing work on the problem of land use plan design.

In general, the probability obtained experimentally of a given plan falling within the predetermined optimal zone was observed to be greater than the theoretical value. This would give an overall indication that the random procedure of module placement can be used with a good degree of success. Apart from the testing of the validity of the random technique, the controlled-experiment procedure was made also to estimate the optimal values of the parameters involving the plan effectiveness. A more detailed description of the experiments and their results is given in the following paragraphs.

Experiment 1: Random Placement With No Constraints

In this series of experiments, the total cost of any one plan is equal to the summation of site and linkage costs. Because each cell was assigned with its own individual soil characteristics, the site cost, which is soil-dependent, was different for each module placement. The cost data were derived from the data used by the Southeastern Wisconsin Regional Planning Commission for the village of Germantown, Wisconsin (5). The study area consisted of 10 cells, each one having an area of 160 acres. Each cell was made 0.5 mile square to aid in defining the locations of the cell centroids, for all linkages are measured from centroid to centroid. Only 5 module types were included in this set of experiments in order to limit the number of iterations in exhaustive search. The module types and their space requirements are as follows:

Type	Number	Acres
Low-density residential	1	126.10
Medium-density residential	2	62.70
Neighborhood park	3	6.40
Neighborhood commercial center	4	64.00
Secondary school	5	20.00

Both the site and linkage costs are given in Table 2. For site costs, the rows are the different cell numbers and the columns are the different module types. Each element of the matrix gives the cost of placing that particular module type in that particular cell. For linkage costs, both the rows and the columns are the various module types. In this case, any one element in the matrix gives the aggregated cost per mile of linking any one module type to another. These linkage costs take into account the cost for connecting module types not only by roads or highways but also by water, sewer, gas, electric, and telephone lines.

Double counting was eliminated in the process of linking module types by a linkage-satisfaction matrix that was set up. This matrix is of the same size as the linkage cost matrix, and it is operated by a simple binary code where 0 means that a particular linkage is not satisfied and 1 means that the linkage is satisfied. The linkage-satisfaction matrix is initialized with all 0's, and it simply becomes a matter of providing linkages until all the elements in the matrix become 1.

The exhaustive-search algorithm was then used to develop all possible experimental plans. Inasmuch as there were 10 cells and 5 modules, the number of combinatorial, nonrepetitive plans was $10 \times 9 \times 8 \times 7 \times 6$ or 30,240 plans. As a part of the output, the total plan costs were printed in descending order. The output data were used to define the optimal zone as well as to obtain the associated plan costs in order to check the results obtained from the random-search algorithm.

To discount any experimental error, we ran the random algorithm several times with different values of a and S. The results obtained from these runs are given in Table 3. These results seem to indicate that the random procedure provides an effective technique when no constraints are assigned with the module-placement process; all the best plans obtained from the random-model runs are well within the specified optimal zone. Therefore, the next step was to determine whether constraints put on the module placement would have any effect on the performance of the technique.

Experiment 2: Random Placement With Positive Constraints

In this experiment, the same set of modules and the same study area as used in experiment 1 were considered, and only positive distance constraints were added to the program. By a positive distance constraint is meant that certain types of modules cannot be farther apart than a specified distance. If, in any experimental plan, the modules are placed farther apart, then the plan is considered infeasible.

The exhaustive-search algorithm was run, and both the feasible and infeasible plan costs were printed out in descending order. The number of feasible plans resulting from the run was 20,880, while the number of infeasible plans was 9,360. This gave a feasibility of 69.05 percent.

Table 1. Plans required to obtain at least 1 plan in optimal zone by random-placement method.

	S						
a	0.80	0.85	0.90	0.925	0.95	0.975	0.99
0.1000	16	18	22	25	29	35	44
0.0750	21	24	30	33	38	47	59
0.0500	32	37	45	50	59	72	90
0.0375	42	50	60	68	78	97	120
0.0250	64	75	91	102	119	146	182
0.0125	128	151	183	206	238	293	366
0.0100	161	189	230	258	299	367	459
0.0075	214	252	306	344	398	490	612
0.0050	322	378	460	517	598	736	919

Table 2. Input data for experiments.

		Experiment 1 and 2 Modules					Experiment 3 Modules			
Item	Cell	1	2	3	4	5	1	2	3	4
Site development cost	1	859,337	1,098,970	124,538	166,663	50,000	859,837	1,098,970	402,204	166,663
	2	753,013	1,002,898	92,871	496,912	50,000	753,014	1,002,899	302,619	496,912
	3	646,190	906,827	63,204	827,160	50,000	646,190	906,827	203,034	827,160
	4	570,050	846,836	60,128	773,276	50,000	570,050	846,835	142,533	773,275
	5	493,910	786,844	75,016	719,390	50,000	493,910	786,844	182,032	719,390
	6	522,857	827,096	55,112	703,690	50,000	522,857	827,096	176,204	703,690
	7	584,524	866,962	59,158	765,425	50,000	584,523	866,962	189,619	765,425
	8	560,819	808,316	59,348	754,155	50,000	560,819	808,216	189,633	754,155
	9	475,448	709,804	55,492	681,150	50,000	475,448	709,804	176,232	681,150
	10	503,273	802,468	59,824	758,950	50,000	508,273	802,468	191,704	788,950
	11						577,231	854,648	197,069	793,055
	12						667,643	904,387	289,218	433,907
	13						684,055	950,719	296,654	462,807
	14						676,874	942,907	292,113	443,027
	15						499,153	768,450	176,218	692,420
Linkage cost	1[a]	0	0	54,800	109,600	13,700	0	0	50,000	109,600
	2[a]	0	0	54,800	109,600	13,700	0	0	50,000	109,600
	3[a]	54,800	54,800	0	0	0	50,000	50,000	0	50,000
	4[a]	109,600	109,600	0	0	0	109,600	109,600	50,000	0
	5[a]	137,000	137,000	0	0	0				
Intermodule distance constraints	1[a]	10.00	10.00	1.75	1.75	4.00	10.00	10.00	-0.70	2.00
	2[a]	10.00	10.00	1.75	1.75	4.00	10.00	10.00	-0.70	2.00
	3[a]	1.75	1.75	10.00	10.00	10.00	-0.70	-0.70	10.00	-0.70
	4[a]	1.75	1.75	10.00	10.00	10.00	2.00	2.00	-0.70	10.00
	5[a]	4.00	4.00	10.00	10.00	10.00				

[a]Modules.

Table 3. Results of random placement.

Constraints	a	S	Number of Trials	Optimal Zone	Rank Number of Selected Plan: Test 1	Test 2	Test 3	Test 4	Test 5	Feasible Plans (percent)
None	0.005	0.99	918	≤151	54	13	15	4	11	
	0.005	0.95	597	≤151	28	20	10	46	109	
	0.005	0.90	459	≤151	26	7	1	82	5	
	0.005	0.80	321	≤151	31	87	2	5	15	
	0.01	0.99	458	≤302	14	14	31	215	1	
	0.01	0.95	298	≤302	82	94	10	44	46	
	0.01	0.90	229	≤302	35	120	137	223	164	
	0.01	0.80	160	≤302	129	196	36	275	40	
Positive (method 1)	0.005	0.99	1,165	≤104	14	3	6	7	21	78.83
	0.005	0.95	765	≤104	41	15	6	101	46	78.10
	0.005	0.90	588	≤104	16	8	34	1	2	78.14
	0.005	0.80	401	≤104	141	31	52	8	22	80.12
	0.01	0.99	584	≤208	23	49	61	14	15	78.49
	0.01	0.95	379	≤208	10	56	23	41	9	78.69
	0.01	0.90	294	≤208	35	95	36	28	72	78.00
	0.01	0.80	201	≤208	18	201	25	205	14	79.72
Positive (method 2)	0.004	0.99	1,178	≤85	87	6	25			78.22
	0.004	0.95	768	≤85	5	67	14			78.16
	0.004	0.90	588	≤85	4	37	39			78.75
	0.004	0.80	411	≤85	16	52	36			78.51
	0.008	0.99	586	≤175	111	62	5			77.26
	0.008	0.95	374	≤175	77	142	31			79.63
	0.008	0.90	296	≤175	38	271	135			77.70
	0.008	0.08	205	≤175	73	84	141			79.03
Positive and negative	0.005	0.99	2,516	≤68	41					36.49
	0.005	0.99	2,574	≤68	2					35.66
	0.005	0.99	2,530	≤68	54					36.28
	0.005	0.99	2,556	≤68	32					35.92
	0.005	0.99	2,368	≤68	4					38.77

The random algorithm was then run a number of times for various values of a and S. The problem of infeasibility of a plan was handled in 2 different ways. In the first case, only the feasible plans were included in the universe of possible experimental plans. An a-value and an S-value were chosen, and an n was calculated from the formula. This n was then the number of feasible plans that must be generated in order to obtain 1 experimental plan that falls within the optimal zone a of the feasible plans. In this procedure, it was not important how many total plans were generated as long as the number of feasible plans equaled n. The results obtained by running the random algorithm using this approach are given in Table 3. Based on the optimal zones as defined by the exhaustive search, the plans selected are well within the desired subregions in almost all cases.

Although the first method of handling the infeasible plans is the most direct and accurate approach, it would require comparatively long computer time as the complexity of the system would increase. Therefore, it was decided to test an approximate approach in dealing with the infeasible plans. In the second method, the infeasible as well as the feasible plans were included in the universe of total plan designs. Accordingly, an adjustment of the a-value, or optimal zone, was necessary and was accomplished in the following manner:

$$a^* = (a)(Pf)$$

where

a^* = probability of obtaining an optimal feasible plan,
a = plan accuracy, and
Pf = probability that a plan is feasible.

The original formula was then modified as

$$S = 1 - (1 - a^*)^n$$

or

$$n = \log(1 - S)/\log(1 - a^*)$$

The occurrence of feasible plans was recorded for some initial iterations, and at some point in the program the percentage of feasible plans Pf was calculated. Because the same level of plan accuracy was desired, that is, an optimal zone of some small percentage of the feasible plans, the original a-value read in as input data was then multiplied by the percentage of feasible plans. The resulting a*-value along with the given probability S was put back into the formula, and a new n or number of experimental plans was calculated. The inclusion of plan constraints has the effect of increasing the number of experimental plans necessary to achieve a given level of plan accuracy. For example, if the specified values of a and S are 0.05 and 0.90 respectively, then the number of experimental plans is 45; whereas, if the plan constraints are considered and the percentage of feasible plans Pf is observed to be 0.10, then the number of experimental plans increases to 460 from 45 to achieve the same plan accuracy.

The results as obtained from the second method are also given in Table 3. The results would again indicate the effectiveness of the random procedure; the rank of the best plan is within the specified optimal zone in almost all trials. Comparing the number of trials required by this approach and those required by the first method shows that they do not vary considerably. However, there is a shortcoming inherent in this method of handling the infeasibility of plans. Because, in this case, the universe of total plans included both feasible and infeasible plans, the modified optimal zone also could contain, by chance, some plans that are not feasible. Furthermore, a comparison of the actual percentage of feasibility with the corresponding percentage values, as obtained in the trials conducted by this method, indicated that these values are on the average 9.36 percent higher than the value calculated by the exhaustive search.

This would mean that the adjusted value of a is larger than it should be, and, therefore, the required trials calculated would be less than they actually should be.

The percentage values of feasible plans as generated by the first method were also computed to check with the results of the second method. In this case, as in the second, the percentage values of feasibility were higher than the actual percentage. Having on the average a value of 9.71 percent higher than the known value would again seem to indicate that a running average from the random procedure would not provide a very reliable estimate of the percentage of feasible plans. It is also felt that this method would decrease in its reliability as the percentage of feasible plans would decrease.

Experiment 3: Random Placement With Positive and Negative Constraints

In this set of experiments a negative distance constraint was added to the constraint schedule. A negative distance constraint means that a certain module type, in this case a sewage-treatment plant, cannot be closer than a specified distance to any of the other module types.

The study area was increased to 15 cells in order to accommodate the negative distance constraint because too small a study area would cause a great percentage of infeasible plans. But the increase in number of cells necessitated a decrease in the number of module types to be considered so that the computer time to conduct an exhaustive search would not be excessive. Accordingly, the number of module types was reduced to 4. Therefore, the number of combinatorial, nonrepetitive plans is $15 \times 14 \times 13 \times 12$ or 32,760. The module types and their space requirements are as follows:

Type	Number	Acres
Low-density residential	1	126.10
Medium-density residential	2	62.70
Sewage-treatment plant	3	45.00
Neighborhood commercial center	4	64.00

An exhaustive search of all possible plans showed that there were 13,664 feasible plans and 19,096 infeasible plans. The percentage of feasibility was therefore 41.71 percent. The random algorithm was run only 5 times with the same values for a and S for each run. The number of trials was calculated on the basis of the feasible plans only. The results of these runs are given in Table 3. In each case, the selected plan fell within the desired optimal zone. Although the number of iterations necessary to generate the required number of feasible plans was high, more than 2,500 in most cases, this number is still small compared with the 32,760 possible plans that would have to be checked if an exhaustive search were to be made.

The percentage of feasibility was also computed for each run. Comparing these values with the known percentage value of 41.71 percent shows that on the average these values are 5.09 percent lower than the known value.

ESTIMATION OF PLAN-EFFECTIVENESS PARAMETERS

When the random method is used, the 2 parameters involving the plan effectiveness must be decided beforehand. These 2 parameters are the desired level of plan accuracy, a, and the probability of success of achieving an optimal plan, S.

Because a land use plan design process is a complex system that entails a large number of factors, an absolute lowest cost plan may not be attainable. Moreover, it can be reasonably assumed that in the total universe of experimental plans there exists a subregion, consisting of a number of plans whose total costs are close to the lowest cost, where the differences in plan costs are not so significant. Accordingly, a satisfactory solution to the problem of optimization can be well achieved if an experimental plan from this subregion is attained. In the random procedure, the relative size of this subregion of good response with respect to the total universe is defined as the plan accuracy. On the other hand, the definition of success used in the procedure is to ob-

tain an experimentally selected plan that falls within the desirable subregion, and the probability of success is the desired level of assurance that a planner would like to have of achieving an optimal land use plan.

To estimate the values of a and S that might be reasonably used in a land use plan design mode, we conducted experiments with various cell-module combinations over a range of values for each of the 2 parameters. The soil characteristics of each cell were also varied to provide different cost values for placing the various module types in different cells. Furthermore, the random-number generators were initialized at the beginning of each trial with different random-number seed values.

First, an exhaustive search procedure was adopted in placing modules in cells, and plan ranks and their costs were obtained in descending order. The algorithm was run several times with changing soil characteristics. The results were then plotted to develop the cost curves. Some of the typical cost curves as obtained from trials with 4 modules and 8 cells are shown in Figure 1. Similar cost curves were also developed for several higher order cell-module combinations, but they are not shown here because of the enormousness of their sizes. It was consistently observed, however, that the assumption of the existence of a low-cost region in the universe of experimental plans is reasonable. Such a subregion or optimal zone is characterized by a valley at the end of the cost curves. This valley region, where the plan costs approximate the lowest cost, is shown in Figure 2, which shows the plan costs of 152 best plans out of a possible 30,400 experimental plans in a set of trials with 6 modules and 10 cells. Each curve shown in Figure 2 represents a trial with different soil characteristics and consequently with different site costs.

It was observed that the significant breaks in cost curves take place at points causing low-cost regions to include, on the average, about 5 to 10 percent of the total plans. Therefore, assumption of a plan accuracy value within a range of 0.5 to 1.0 percent would be very reasonable. Lower values would be desirable for plan design of small areas, such as in site planning or for those situations where cost estimates are more precise and accurate and where a comparatively small difference in plan cost could affect the planning decisions. On the other hand, higher values of plan accuracy would be reasonable for the design problem of large areas, such as a region where the model input data cannot be so precise and the planning decisions are dependent on many factors.

In the next set of trials, results from the random algorithm were compared with the exhaustive search output to obtain a distribution of best plan rank. In each trial 50 runs were made for each a and S combination, and the rank of the best plan obtained from a random run was noted. The ranking was done on the basis of the exhaustive search output. The results obtained from the trials made with 4 modules and 10 cells are given in Table 4, where the distribution of best plan rank is presented for various combinations of a- and S-values. The best plan rank numbers for only 95th percentile and lower values are shown because for higher percentile values the best plan rank numbers, in some cases, fell beyond the optimal zone. The percentile value indicates the probability that the best plan obtained from a particular random run will be at least the xth lowest cost plan. For example, the probability that the best plan generated by a random run with a-value of 0.005 and S-value of 0.99 will be the fourth lowest cost plan or better is 0.80.

It may be observed from the entries given in Table 4 that, for a given value of plan accuracy, the best plan rank number seems to become better as the S-value for the probability of plan success is made higher. However, although the S-value of 0.9 gave significantly better results than the S-value of 0.8, the results did not improve appreciably as the S-value was increased to values higher than 0.9. This trend was consistent for all the percentile values of best plan rank number distribution. Therefore, it would seem to be reasonable to assume that a range of 0.85 to 0.95 is an appropriate range of values for S. Because an increase in the S-value would mean an increase in number of iterations, the S-value could be taken as 0.9 in most cases without affecting the plan results. However, higher values can be considered in such cases where cell-module combinations are not large, and lower values would be reasonable for a high order of cell-module combinations.

Figure 1. Costs of plans with 4 modules and 8 cells.

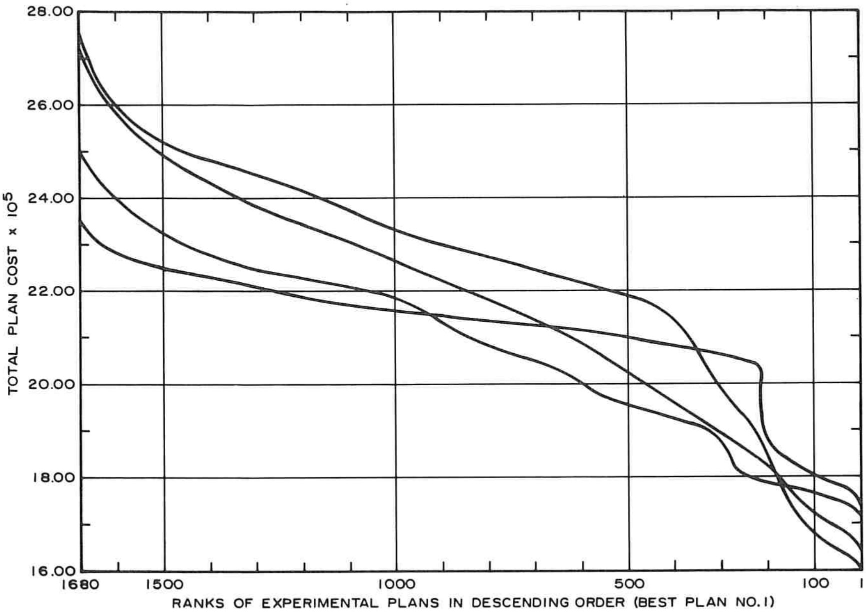

Figure 2. Optimal zones of costs for plans with 6 modules and 10 cells.

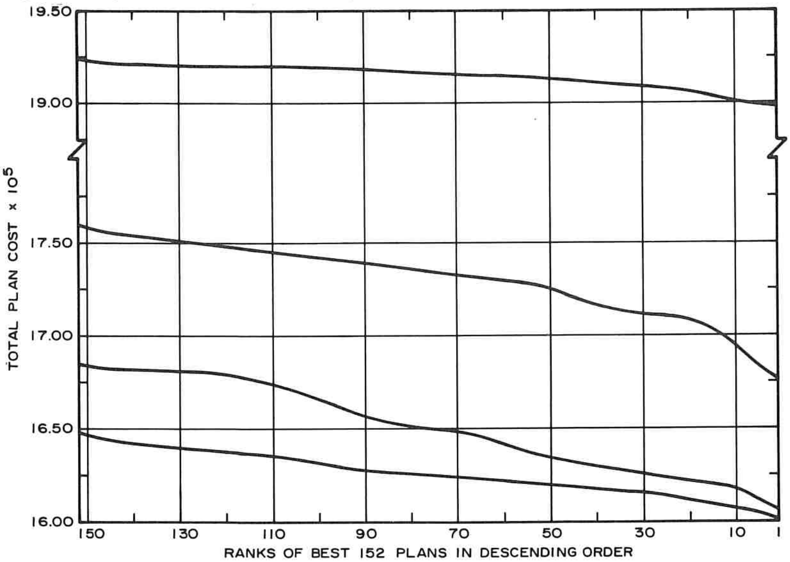

Table 4. Distribution of best plan rank.

		Percentile						
a	S	0.25	0.50	0.60	0.70	0.80	0.90	0.95
0.005[a]	0.99	1	2	3	3	4	7	9
	0.95	1	3	3	6	9	11	14
	0.90	3	5	6	8	12	16	20
	0.80	3	5	8	11	17	22	25
0.01[b]	0.99	3	5	6	8	12	16	20
	0.95	3	5	8	11	17	22	25
	0.90	3	9	12	14	18	25	33
	0.80	4	16	21	23	28	37	38
0.025[c]	0.99	5	10	13	16	23	30	36
	0.95	7	24	27	35	50	80	100
	0.90	6	21	27	38	58	90	97
	0.80	11	38	47	57	70	94	120

[a]Optimal zone, 25 lowest cost plans.
[b]Optimal zone, 50 lowest cost plans.
[c]Optimal zone, 125 lowest cost plans.

CONCLUSIONS

A land use plan design nolds immense potentiality in the field of land use planning. The model can be used to design a set of ideal plans for a series of forecast years ranging from 5 to 30, each design being developed independent of others and based only on the initial conditions and the forecast requirements. The series of land use plan designs derived from the model will then display the most economic and efficient land use pattern that can be obtained at a particular design year. This, in turn, will aid in making decisions concerning the development of public and private policies regarding the use of land in a systematic and efficient way.

The model provides a normative tool rather than an evaluation process to arrive at an optimal land use plan design. While considering the role of the plan constraints in shaping the final solution, one should make a distinction between values and a design to satisfy values. The search procedure used in the model attempts to create a design, and the plan constraints are simply the limitations imposed on such a design. The model does assume a set of values that is reflected in a set of plan constraints. Because values are subjective, there can be no optimal set of values. The model is therefore a normative one given a set of values. The same statement can be made of any normative model.

The basic use of the proposed model is in preparing a target plan at any level of land use planning, such as neighborhood, community (town or village), metropolitan, regional, state, or even national level. Although at any of these levels the general structure of the model algorithm remains the same, the nature of the input data and plan constraints would be different from one level to another. The type of input data currently available makes it possible to apply the design model only at community, metropolitan, and regional levels. Furthermore, the model can be well utilized in capital works programming in a time-simulation framework. By running a series of design model runs on a 5-year time increment starting back from the target year, the proper sequence of capital works programming can be determined. The greatest impact of the proposed model on metropolitan plan-making will probably be in establishing a standard or norm against which all proposed plans can be evaluated. Another application relates to measuring the cost of any suggested plan design constraints.

The use of a simple procedure for improving the operation of the model based on the random-search technique has been discussed here. The validity of the random technique was established through a series of controlled experiments where the results obtained from the random model algorithm were compared with the results generated by an algorithm based on an exhaustive-search technique. The study clearly indicated that the random method of module placement can be used with a good degree of success. Apart from the testing of the validity of the random technique, the controlled experiment procedure was also used to determine the appropriate values of the parameters involving the plan effectiveness.

It was the intent of the paper to evaluate the merit of the random-search technique as a useful tool in the preparation of a land use plan design. This has been established through the result of small-scale controlled experiments with hypothetical areas. However, the random algorithm is currently being applied in several real-world problems of land use plan design at different levels of planning in the southeastern Wisconsin region.

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